The Carbon Dating Problem for the Shroud of Turin, Part 2: Statistical Analysis

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Abstract

In 1988, the C^{14} dating methodology was used (Damon, et al., Ref. 1) to date samples from the Shroud of Turin to 1260 to 1390 AD. But research during the last 30 years has convinced leading Shroud researchers that the Shroud of Turin is much older than 1260 to 1390 AD, thus contradicting the results of the C^{14} dating. To solve this carbon dating problem for the Shroud of Turin, a three-part series has been written that covers: 1) background, 2) statistical analysis, and 3) the neutron absorption hypothesis. Part 1 on background information (Ref. 2) should be read and understood before this paper is attempted. This paper is part 2 in the series and discusses the statistical analysis of the 1988 C^{14} date measurements. To the extent possible, this paper is written for the layman, and uses only the Chi-squared statistical analysis technique that was used in Damon. Part 3 in this series (Ref. 3) hypothesizes that neutron absorption in the Shroud explains the results found in the statistical analysis in this paper, as well as the C^{14} date to 1260 to 1390 AD. In this paper, several indications are discussed that the range of the 1988 C^{14} date measurements is much too large in comparison to the measurement uncertainties. This indicates that there is a serious problem with the data. Most significantly, a Chi-squared statistical analysis of the measurement data indicates that the variation in the C^{14} date measurements has only a 1.4% probability of being consistent with the measurement uncertainties. This indicates a 98% probability that something other than random measurement error was also affecting the measured values, such as, in statistical analysis terminology, a systematic bias. Plotting the three laboratory average values indicates that this systematic bias is a function of the distance of the sample from the bottom of the Shroud. This means that each measured value M_i was the result of the actual age (A) of the Shroud plus the bias $B(x_i, y_i)$, where the x_i and y_i are the prior position coordinates of each sample (i) on the Shroud, combined with the one sigma random measurement error R_i. The equation for this is $M_i = A + B(x_i, y_i) \pm R_i$. This systematic bias could have been sufficient to shift the date for the Shroud forward by up to thousands of years. Ref. 3 discusses the cause for this systematic bias. Since the measured values listed in Damon are not corrected for the magnitude of the bias, the 1988 C^{14} date for the Shroud of 1260 to 1390 AD should not be accepted as necessarily valid, so that the conclusion that the Shroud dates it the Middle Ages is not proven.

1. Introduction

A burial cloth called the Shroud of Turin has been in Turin, Italy, since 1578. Ancient tradition claims that this burial cloth is the authentic burial cloth of Jesus of Nazareth. Amazingly, on the Shroud of Turin can be seen the front and back images of a naked man that was crucified exactly as the New Testament says that Jesus was crucified (Figure 1 of Ref. 2). To determine whether the Shroud of Turin could be the authentic burial cloth of Jesus, more research has been done on the Shroud than on any other ancient artifact (Ref. 4 to 7). Scientific research on the Shroud during the 90-year period between 1898 and 1988 increasingly supported the authenticity of the Shroud, but the C^{14} dating of the Shroud in 1988 produced a date range of 1260 to 1390 AD, with a stated 95% probability that the true value is within this range. The 1988 measurement results including the initial statistical analysis of the results was published in the British journal "Nature" on February 16 of 1989 (Ref. 1). This paper is often referred to as "Damon" because he was the first author listed.

With many years of additional research on the Shroud since 1988, most leading Shroud researchers have concluded that the interpretation of the C^{14} measurement data was badly flawed since there are several lines of evidence that the Shroud existed long before 1260 AD (Sections 4, 5, and 6 of Ref. 2). Multiple Shroud researchers have documented their statistical analysis of the 1988 measurement data (Ref. 8 to 15) with their conclusion that the 1988 measurements are not consistent with the measurement uncertainties, so that the measurements must have been altered by something, so that they should not be accepted as necessarily accurate. The statistical analysis of the 1988 measurement data is continued in this paper to check previous analysis results and to expand the extent of the previous analysis. The statistical analysis of the C^{14} dating results in this paper assumes that the individual measurement values and their uncertainties reported in Damon are based on correct measurement of the C^{14} in the samples. But it is not assumed that the Shroud was made in the Middle Ages, so that it could be Jesus' burial cloth, so that the measured values could have been affected by a systematic bias resulting from unique events at the end of Jesus' life (Section 5 of Ref. 2). The nature of a systematic bias is discussed in Section 7 of Ref. 2.

2. Results of the C^{14} Dating of the Shroud in 1988

With the authenticity of the Shroud appearing to be increasingly probable in the 1980s, it was decided to date the Shroud using the C^{14} dating methodology, which is explained in Sections 2 and 3 of Ref. 2. In this dating methodology, the C^{14} isotope is measured in comparison to the C^{12} and C^{13} isotopes because C^{14} decays with a 5730-year half-life, as shown in Figure 1, whereas C^{12} and C^{13} are stable. For this purpose, on April 21 of 1988, samples were cut from the lower corner of the Shroud as shown in Figure 2 and sent to three laboratories for C^{14} dating: the dating laboratory at Oxford in England, Zurich in Switzerland, and Tucson in Arizona. Oxford and Zurich were each sent one sample. Tucson was sent two samples as shown in Figure 2 so that each laboratory would receive about the same total weight of sample material, i.e. about 50 mg each. Tucson did its C^{14} dating using only subsamples from what is called sample A1 in Figure 2. The smaller sample A2 that was sent to Tucson was put into a safe in Tucson without being dated. Thus, it is believed that the dating laboratory at the University of Arizona (http://www.physics.arizona.edu/ams/index2011c.htm) in Tucson now has the remainder of sample A1 and all of sample A2 in two different safes. It is not entirely clear whether the laboratories in Oxford and Zurich have retained any of the remaining material from the samples that were sent to them. Information regarding how the samples were subsampled at the three laboratories has not been made available and may not have been recorded.

Table 1 summarizes the values listed in Damon. Material 1 was the samples from the Shroud of Turin. Samples from three other sources of known age were used as standards by the three laboratories for comparison of results. These three other sources are labeled in Table 1 as materials 2, 3, and 4. Vertically, Tucson is listed toward the top with Zurich below it, with Oxford below Zurich. This sequence is followed throughout this paper because this is the sequence that the samples were cut from the Shroud as shown in Figure 2. Of the samples that were dated, the sample sent to Oxford was furthest to the left, the sample sent to Zurich was in the middle, and sample A1 sent to Tucson was furthest to the right.

Table 1 lists the dates in terms of the years before present (YBP) where the reference year for the "present" is 1950. In Table 2, the corresponding year AD is given for the Shroud samples. Those doing the statistical analysis in Damon chose the "unweighted mean" of 1259 ± 31 in Table 2 over the weighted mean of 1261 ± 16. The unweighted mean of 1259 was then rounded off to 1260 AD. This is the "uncorrected value". When this uncorrected value is corrected for the variable amount of C^{14} in the atmosphere at ground level, a date range of 1260 to 1390 AD is obtained. This is a two-sigma date range which means that there ought to be a 95% probability that the true value falls within this range, if the statistical analysis is done correctly. It is this range of dates and this apparent probability that prompted the scientists to conclude and the media to report in 1989 that the Shroud of Turin had been scientifically proven to be from the Middle Ages, and thus could not possibly be the authentic burial cloth of Jesus. However, many questions arise over the adequacy of the statistical analysis of the measurement data. The procedures and problems related to the dating of the Shroud are summarized in Chapter 8 of Ref. 6, Chapter 14 of Ref. 7, and Ref. 19.

3. Basis and Conclusions of the Statistical Analysis

Most people familiar with the dating results discussed in the previous section (1260 to 1390 with a 95% probability) probably believe that this is the end of the story, but it's not. But before we discuss the details of the statistical analysis of the measurement results, the basis for the analysis and the conclusions of the analysis will be summarized. This will allow those who are not skilled in statistical analysis to benefit from this paper.

The basis for the statistical analysis discussed below is the following:

- The C^{14} dates for each measurement reported in Damon are assumed to be based on accurate measurements of the quantity of C^{14} relative to C^{12} and C^{13} in each subsample. The three laboratories used progressive cleaning of the subsamples, but according to paragraph 21 of Damon this did not have a significant effect on the C^{14} dates: "From these data it can be seen that, for each laboratory, there are no significant differences between the results obtained with the different cleaning procedures that each used." While it may be possible to challenge this statement, for simplicity and consistency with Damon, it will be assumed to be true in the analysis in this paper. Thus, the measured values are assumed to not be significantly affected by cleaning procedures or contaminants. It was also assumed that measurement of the samples was not affected by bacteria, isotopic fractionalization, or by patches or reweaves of the fabric (Section 2 in Ref. 3 and Chapter 9 in Ref. 7).
- Any systematic bias related to the standards and the C^{13} correction are assumed to be properly factored into the C^{14} dates and/or their uncertainties reported in Damon.
- All random error due to measurement of standards and the C^{13} correction are assumed to be properly factored into the measurement uncertainties reported in Damon, based on their statement that "The errors, which are quoted in Table 1 at the 1*sd* level (*sd* is standard deviation), include the statistical (counting) error, the scatter of results for

standards and blanks, and the uncertainty in the Ó^{13}C determination (Arizona includes the Ó^{13}C error at a later stage, when combining subsample results; Oxford errors below 40 yr are rounded up to 40)." (Damon, paragraph 20) Thus, the uncertainty assigned to each measured value is assumed to be accurate based on the laboratories' determination of the best values for the random measurement error. These uncertainty values based on the random measurement error will not simply be assumed to be underestimated, as was done in Damon. But another potential cause of measurement variation will be considered, e.g. due to systematic bias. This possibility is not considered in Damon.

- Given the above assumptions, a χ^2 (Chi squared) statistical analysis of the data is a correct methodology to determine whether the variation in the measurements is due to only random measurement error or whether a systematic bias is also present. This is the methodology used in Damon. For consistency with Damon, this methodology will be the only statistical analysis technique used in this paper.
- It is assumed that no intentional deception or deceit has occurred in the sampling, measurement, or reporting of the measurement results. But aspects of the statistical analysis are questionable.

The conclusions of the statistical analysis discussed in this paper are the following:

1. The laboratory in Tucson made eight measurements on the Shroud samples (Table 4) instead of the four measurements listed in Damon (Table 1). This reduction from 8 to 4 measurements was accomplished by averaging pairs of values. This process eliminated Tucson's highest and lowest values from the report so that the measurement data listed in Damon appeared to be more consistent than it was. It was many years before it was revealed that Tucson had done eight measurements instead of just four.
2. Some of the statistical analysis values reported in Damon are significantly different than the values calculated in this document (Table 6 vs. Table 1). These are compared in Table 9. Where these differences occur, it is often not clear how the values in Damon were obtained, so that they cannot be confirmed to be correct.
3. It was assumed in Damon that the variation in the measurements was only due to random measurement errors. But the Shroud may have been involved in unique events that could have resulted in a systematic bias. For example, if the Shroud is Jesus' burial cloth as ancient tradition maintains, and if Jesus' body disappeared from within the Shroud as the earliest historical documents maintain, then this would have been an event that is beyond or outside of our current understanding of the laws of physics. And if the disappearance of his body was a real historical event, as Christians believe, then we would have no idea what other phenomena could have accompanied such an event. Specifically, the disappearance of his body from within the Shroud could have been accompanied by a burst of radiation that could have included neutrons, which would have created new C^{14} on the Shroud. This would shift the C^{14} date in the forward direction, as discussed in Ref. 3.
4. When a chi-squared statistical analysis is applied to all 12 or 16 measured values in Table 5 or Table 6, with the laboratory's assumption that all variation in the measurements is only due to random measurement error with no systematic bias, the result is a significance level of only 1.4% (bottom line of Table 5 or 6). This means that there is only a 1.4% chance that the

spread in the measured values is consistent with the stated measurement uncertainties. This falls well below the usual minimum acceptance criteria of 5% and implies that with a high probability (98%) something was affecting the measured C^{14} dates, causing them to be different from the true dates, such as, in the terminology of statistical analysis, a systematic bias. Thus, with about a 98% probability, there could have been a systematic bias that would have changed the measured C^{14} dates from their true values to incorrect apparent values. In Damon, this systematic bias was not recognized nor was the magnitude of it determined, so that the measured values were not corrected for it. This means that the C^{14} date for the Shroud in Damon (1260 to 1390 AD) should not be accepted as necessarily valid. It is not justified to simply assume that the C^{14} measurement uncertainties were underpredicted, as was done in Damon, because the measurement uncertainties were determined using the same equipment and procedures as the C^{14} measurements. In other words, if it is valid to assume that the uncertainties are wrong, then it is probably as valid to assume that the measurements are wrong for they would have both been determined using the same equipment and procedures. All significance levels in this paper were obtained from the website http://www.socscistatistics.com/pvalues/chidistribution.aspx using two degrees of freedom.

5. The C^{14} dates from the three laboratories are not statistically consistent with each other. When all 16 measured values are included, the average values are Oxford = 1200.8 ± 30.7, Zurich = 1273.9 ± 23.7 and Tucson = 1303.5 ± 17.2. The average values from the laboratories in Oxford and Tucson are statistically different. The difference, including a statistical calculation of the uncertainty, is 102.7 ± 35.2 (1303.5 – 1200.8 = 102.7 and the square root of $17.2^2 + 30.7^2 = 35.2$). This is nearly a three-sigma difference (102.7 / 35.2 = 2.92), which is outside of the normal acceptance criteria of two sigma. This indicates that the samples sent to Oxford and Tucson are likely to contain different amounts of C^{14}. But how could they contain different amounts of C^{14} when they were both cut from the same area of the Shroud? According to the neutron absorption hypothesis (Ref. 3), the explanation is that the samples sent to Tucson and Oxford were basically different because they had different amounts of new C^{14} produced in them because of the neutron distribution in the tomb, which would have naturally taken a cosine shaped distribution in the limestone tomb (Figure 9 in Ref. 3).

6. If the above average values from the three laboratories are plotted as a function of the distance from the end of the Shroud, a reasonably linear plot is obtained with a slope or gradient of about 36 years per cm (Figure 3). This means that if the sample point is moved one cm closer to or further from the edge of the Shroud then the measurement of the C^{14} age will change by about 36 years. If the sample point is moved 1.0 inch (2.54 cm) then the age will change by about 91 years. And if, assuming the extrapolation to be accurate, the sample point is moved 13.5 inches (34.2 cm) then the age will change by about 1230 years which is the difference between the time of Jesus and the lower limit obtained from the C^{14} dating (1260 AD). This indicates that the C^{14} date that was measured for each sample was different because of the location that it had occupied on the Shroud. Though the three samples were located next to each other on the Shroud, there was enough difference in the C^{14} content at the different sample locations to affect the ratio of C^{14} to C^{12} and C^{13}. But what could cause this slope of about 36 years per cm in the C^{14} date at the sample location? Nuclear analysis computer calculations (Ref. 3) obtain a similar slope for the C^{14} date depending on how the Shroud was folded at the feet, due to the normal distribution that neutrons take in a limestone tomb as it would have been constructed in Jerusalem in the first century.

In conclusion, extensive statistical analysis of the C^{14} measurement values by multiple experts (Ref. 8 to15) indicates that the variation in the measured values was not only due to random measurement errors, which are common to all measurements, as assumed by the three laboratories, but also due to a systematic bias that would have affected all the measurements. This systematic bias, since it was not identified and quantified so that the measured values could be corrected, indicates that the C^{14} dates in Damon should not be accepted as necessarily valid. This raises the question about what could have caused a systematic bias sufficient to result in a date to the Middle ages. According to the neutron absorption hypothesis (Ref. 3), the answer is neutron absorption creating new C^{14} atoms on the Shroud.

4. Statistical Analysis in Damon

Sixteen date measurements were made on subsamples by the three laboratories. The statistical analysis of this data was reported in Damon (Ref. 1). This statistical analysis is questionable from many perspectives.

In repeated measurements of a physical quantity, there will normally be some variation in the measured values. This variation can be due to either random measurement errors or a systematic bias. Since random errors might cause a measured value to be a little high one time and a little low another time, these variations from the true value will mostly cancel when the average is calculated for many measurements. But a systematic bias, since it is not random but is a function of some parameter (prior position of the sample on the Shroud in the case of the C^{14} dating of the Shroud), can cause the average of the measured values to be significantly displaced from the true value.

When the C^{14} dating of the Shroud was done in 1988, it was well known that the continuous history of the Shroud only went back to when it was exhibited in Lirey, France, in about 1356, but the many evidences that the Shroud's history went back several centuries before the 14^{th} century were not well known. As a result, those doing the statistical analysis of the 1988 measurement data probably would have assumed that the Shroud was from the Middle Ages, so that there was nothing unusual about the Shroud, so that it could be dated by the C^{14} dating methodology as any other piece of fabric. This belief would have led to an assumption that the variations in the measurements would only be due to random errors (Damon, paragraph 22) and not also due to a systematic bias. But in their statistical analysis they found that this basic assumption was not true (Damon, paragraph 22): "The underlying principle of the statistical analysis has been to assume that … the quoted errors fully reflect all sources of error …" with the "quoted errors", i.e. measurement errors stated in Damon, being calculated only from random error components. But it was concluded in Damon paragraph 23 that "it is unlikely that the errors quoted by the laboratories … fully reflect the overall scatter." This is important because it shows that those doing the statistical analysis in Damon recognized that it was very likely that the measured values varied more than would be caused by the stated measurement uncertainties alone. This indicates that something strange was going on that they did not understand, such as the presence of a systematic bias that could have caused all measurements to be off. Thus, the resulting average of 1260 AD (uncorrected) should not be accepted as necessarily valid. But instead of recognizing this, just the opposite was concluded: "These results provide conclusive evidence that the linen of the Shroud of Turin is mediaeval" (paragraphs 1 and 28 of Damon). Thus, this statement results from a misinterpretation of the measurement data that resulted in a failure to recognize the evidence that a systematic bias had affected measurements at all the laboratories. The reason for these problems in their statistical analysis appears to be that they could not conceive of any reason for a systematic bias to be affecting the measurements, and this could have resulted from an assumption that the Shroud was a forgery from the Middle Ages and thus not related to Jesus or the reported disappearance of his body from within the Shroud.

The values reported in Damon for the statistical analysis of the measurement data are listed at the bottom of Table 1. Damon reports that "From these data can be seen that, for each laboratory, there are no significant differences between the results obtained with the different cleaning procedures that each used" (Damon, paragraph 21). This statement cannot be statistically confirmed from the data since in many cases there are only single observations of a particular cleaning procedure, so Damon overstates the case. The seven different precleaning procedures actually confound the statistical analysis making it even murkier. But for simplicity and consistency with Damon, it will be assumed to be true, i.e. that progressive cleaning makes no difference. This eliminates the need to consider normal contamination such as wax, oils, talc, etc., which would be removed by the normal processes used to clean the samples, and it eliminates the need to account for the effect of the various cleaning methods on the measurement results. What the layman calls an "average" value is called a "mean" in statistical analysis. The "weighted mean" for each laboratory, or the grand or final mean for all laboratories, are calculated using a weighting factor that is the inverse of the square of the one sigma uncertainty that is specified for each value. Use of this formula assumes that the stated uncertainty consists entirely of random errors, thus not allowing for a systematic bias. Damon paragraph 22 says, "The underlying principle of the statistical analysis has been to assume that, unless there is strong evidence otherwise, the quoted errors fully reflect all sources of error and that weighted means are therefore appropriate." As will be shown below, there is "strong evidence otherwise" so that "the quoted errors" do not "fully reflect all sources of error" because besides variation due to the random measurement error there is also a systematic bias related to the original position of the sample on the Shroud.

Consider the last two lines of Table 1, where the results of a statistical analysis technique called a χ^2 (chi squared) analysis are listed. This analysis is done to determine the probability that the scatter or difference among the three mean values from the three laboratories (646, 676, and 750 years before present for material 1) could be consistent with the stated uncertainties for these values ($\pm$ 31, $\pm$ 24, and $\pm$ 30). These uncertainties were stated in Damon as though they were derived from the individual measurement uncertainties, assuming these stated uncertainties are only due to the random variation in the measurements. Using the calculated χ^2 value of 6.4 in Table 1, the significance level is listed as 5% on the last line. The significance level is the probability of obtaining by chance the degree of scatter (from a low of 646 to a high of 750 years before present for material 1) among the three mean values (646, 676, and 750 for material 1) from random measurement error alone assuming the stated uncertainties reflect all sources of variation. For materials 2, 3, and 4, the significance levels in Table 1 are 90%, 50%, and 30%. This means that for materials 2, 3, and 4 the scatter in the mean values between the three laboratories (for example 927, 941, and 940 for material 2) is consistent with the stated measurement uncertainties assuming the only variation is due to random variations in the measurements. This means that for materials 2, 3, and 4 the spread in the laboratory means is probably consistent with the measurement uncertainties. (Note: Heteroskedascity, where different sub-populations have different variabilities, could be relevant in material 3 and should be tested for.) But for material 1, which is the Shroud of Turin, the stated significance level in Table 1 is only 5%. This means that there is only a 5% chance that the scatter of mean values reported by the three laboratories for the Shroud samples (646, 676, and 750 for material 1) could result from random variations in the measurement process. This means that there is probably about a 95% probability that either the random variations from the measurement process are significantly underestimated, or that there is something causing a systematic change in the C^{14} dates for example as the result of the original location of the sample on the Shroud. But the random variations from the measurement process were not significantly underestimated for materials 2, 3, and 4, so it is unlikely that they were significantly underestimated for material 1. This leaves only the alternative, i.e. that there is a very good chance (95% probability) that there is something causing a systematic change in the C^{14} measurements for the Shroud.

That the analysts in Damon recognized the issue and how they responded to it is indicated in paragraph 23 of Damon. "More quantitatively, to establish whether the scatter among the three laboratory means was consistent with their quoted errors, a χ^2 test was applied to the dates for each sample, in accordance with the recommended procedure of Ward and Wilson[13]. The results of this test, given in Table 2 (of Damon), show that it is unlikely that the errors quoted by the laboratories for sample 1 (samples from the Shroud) fully reflect the overall scatter. The errors might still reflect the uncertainties in the three dates relative to one another, but in the absence of direct evidence on this, it was decided to give the three dates for sample 1 equal weight in determining the final mean, and to estimate the uncertainty in that mean from the scatter of results." This is indicated in Table 1 by the bold values. For materials 2, 3, and 4, the "weighted mean of the weighted means" is bolded, indicating that 937 ± 16, 1964 ± 20, and 724 ± 20 are used for the final mean for materials 2, 3, and 4 respectively. But for material 1, the "unweighted mean of the weighted means" is bolded, indicating that 691 ± 31 is used for the final mean rather than 689 ± 16, which is the "weighted mean of the weighted means".

The reference to "Ward and Wilson[13]" refers to Ref. 13 in Damon, which is "Ward, G. K. & Wilson, S. R. Archaeometry 20, 19-31 (1978)". This refers to a statistical analysis technique in which a weighted mean is calculated by weighting the individual values by the inverse of the square of their uncertainties. The comments in parentheses are added for clarification, i.e. what is referred to as "sample 1" in Damon is referred to as "material 1" in Table 1, because there are various samples for each material. "The results of this test, given in Table 2" refers to Table 2 in Damon. Results from Damon's Table 2 are listed at the bottom of Table 1 in this paper. The following can be learned from the above paragraph:

- The "significance level" for the Shroud samples (bottom line of Table 1) is 5%, which means that there is only a 5% chance that the spread in the laboratory means is consistent with the stated uncertainties based on the random measurement errors alone. Based on this, the analysts concluded "that it is unlikely that the errors quoted by the laboratories for sample 1 [material 1 = Shroud samples] fully reflect the overall scatter." Thus, they assumed that the low significance level of only 5% was the result of the random measurement error being underestimated, but they offer no evidence for this and there is another option. The other option is that there could be something causing a systematic bias in the measurements. And of these two options, the first option (underestimation of the random errors) should be judged to be less than likely because the estimated random errors for materials 2, 3 and 4 are apparently reasonable because the significance levels for these materials are 90%, 50%, and 30% in Table 1.

- The standard method of doing a statistical analysis of this kind is to calculate the final or grand mean by weighting the laboratory means by the inverse of the square of the uncertainties. This process was used for materials 2, 3, and 4 because the relatively high significance levels (90%, 50%, and 30%) for these materials indicated that the spread in the laboratory means was reasonably consistent with the stated measurement uncertainties. But this was not the case for material 1 because the low significance level of only 5% in Table 1 indicated that there was something inconsistent about the data for material 1. The analysts should have considered the possibility of something causing a systematic change in the measurement data, for example based on the original location of the sample on the Shroud. But instead, the analysts simply assumed that the random errors had been underestimated, and thus rejected the weighted mean (689 ± 16 in Table 1 for material 1 which is the Shroud) in favor of the unweighted mean (691 ± 31) with the uncertainty (± 31) calculated from the spread of the laboratory means (646, 676, and 750). This is the assumption that led to the uncorrected date of 1260 ± 31 AD, which led to the corrected date range of 1260 to 1390 AD with a 95% probability.

The analysts chose this methodology of dealing with the failure of the χ^2 test for material 1 (Shroud samples) because their stated assumption was that the uncertainty would be entirely due to random measurement errors rather than to allow for the possibility that there could be something causing a systematic bias in the measurement values. Perhaps they made this assumption because they were convinced that the Shroud was from the Middle Ages, based on it being exhibited in France in about 1356. To assume that all errors must be random errors is to risk masking a systematic bias that could cause an incorrect conclusion to be drawn from the data. This probably is the reason that samples from the Shroud in 1988 were C^{14} dated to 1260 to 1390 AD when there is so much evidence that the Shroud could not date to the Middle Ages (Section 6 of Ref. 2).

5. Recalculation of the Statistical Analysis

In Table 3, the last four rows in Table 1 were recalculated using an EXCEL spreadsheet starting from the laboratory mean values, for example 646 ± 31, 676 ± 24, and 750 ± 30 for the Shroud samples. The results are listed in Table 3 with two additional digits to facilitate comparison and accuracy for rounding, but the reader should realize that these two additional digits may not be statistically significant. The unweighted mean and the weighted mean recalculated in Table 3 round off to the values in Table 1, which confirms that the correct methodology is being used to obtain these values. But the values recalculated for the χ^2 and the significance level in Table 3 are not the same as the values in Table 1. Apparently the χ^2 and the significance levels for materials 2, 3, and 4 in Damon are significantly rounded values. For example, for material 2 the

χ^2 was rounded from 0.138 in Table 3 to 0.1 in Table 1, and the significance level was rounded from 93.4% to 90%. And for material 1 (samples from the Shroud), the χ^2 was rounded from 6.35% to 6.4%, but the significance level of 4.18% in Table 3 was rounded up to 5% in Table 1, which is from Table 2 in Damon.

This low significance level of 4.18% in Table 3 resulted from use of the exact laboratory mean values reported in Damon (646.00, 676.00, and 750.00). These values are undoubtedly rounded values. To determine the effect of this rounding process on the significance level, the following analysis was performed. The significance level is the probability that the range of the measured values could result from a random variation of the measurements given their stated uncertainties. Thus, the largest significance level will be produced when the difference between the smallest and the largest measured values is minimized. This occurs when the measured values are assumed to be 646.49, 676.00, and 749.50 before rounding. These values round to the values in Damon. With these assumed measurement values, the $\chi^2 = 6.2407$ which produces a significance level = 4.41% for two degrees of freedom. Thus, even under this most conservative assumption, the significance level should still have been rounded down to 4% rather than up to 5%. Why would it be rounded up to 5% in Table 2 of Damon when this was contrary to the normal rules for rounding? The normal acceptance criterion is 5%. This means that if there is at least a 5% chance that the range of the measured values is due to random variations in the measurements, based on the stated measurement uncertainties, then it should be regarded as possible for this range to result from random measurement variations alone. At less than 5%, the range of the measured values should be regarded as probably not due to random measurement variations alone, so that something else such as a systematic bias is probably also affecting the measurements. This provides a motivation to obtain at least a significance level of 5% in the statistical analysis. Listing a value of 4% for this value in Table 2 of Damon would have indicated that the data was probably inconsistent, possible due to a systematic bias in the measurements, thus suggesting that the measured values were not necessarily valid.

In Table 3, the last four rows in Table 1 were recalculated starting from the laboratory mean values, for example 646 ± 31, 676 ± 24, and 750 ± 30 for the Shroud samples. In Table 5, this recalculation starts from further back in the calculational sequence, starting from the 12 measured values, each with its uncertainty, that were listed in Damon as listed in Table 1. In comparing the results in Table 5 with the values in Table 1, the laboratory means are in good agreement with Damon's values (646.44 vs. 646, 676.14 vs. 676, and 749.17 vs. 750), and the uncertainties for the means are in good agreement for Zurich and Oxford (23.74 vs. 24, and 30.70 vs. 30), but the uncertainty for the mean for Tucson is significantly different (17.05 vs. 31). The uncertainty for the mean for Tucson of 31, as stated in Damon and in Table 1, does not appear to have a correct calculational basis. Using the correct value of 17.05 rather than 31 causes the χ^2 to increase from 6.35 in Table 3 to 8.60 in Table 5. This causes the significance level to decrease from 4.18 in Table 3 to 1.40 in Table 5. This means that there is only a 1.4% probability that the range of the measured values could result from a random variation of the measurements given their stated uncertainties. This implies about a 98% probability that something other than random variation is affecting the measurements, such as a systematic bias. It should be noted that if there is a systematic bias present, this process of estimating the uncertainty in the mean from the scatter of the results will mask the presence of any significant systematic bias, and thus result in a failure to recognize that the measurements have been altered from their true values by the systematic bias.

Based on what is said in Damon, it was long believed that the dating laboratory in Tucson in 1988 had performed only four measurements on the Shroud samples. But years later it was discovered that the laboratory in Tucson had made eight measurements on the Shroud samples as listed in Table 4 (p. 36 of Ref. 10 and p. 270 of Ref. 11). The eight measurements were collapsed into the four values reported in Damon by combining pairs of values that were obtained on the same day, as shown in Table 4. When pairs of values in the second column of Table 4 are weighted by the inverse of the square of the one sigma uncertainty in the second column, then the weighted means for the four values are obtained as shown in the third column of Table 4. And when the one sigma uncertainties in the second column are statistically combined by the inverse square of their values, the weighted one sigma uncertainty values shown in the fourth column were obtained. The point is that, when rounded off, this process correctly produced the values and their one sigma uncertainties that were published in Damon, as shown in the fifth column of Table 4.

In Table 6, the values are recalculated starting from the measured values reported in Damon but including all eight measured values from Table 4. The weighted means for each laboratory were calculated using the standard procedure of weighting the values by the inverse of the uncertainty squared. There are some significant differences between the values reported in Damon as listed in Table 1 and the recalculated values listed in Table 6. If the values of the measurement uncertainties stated in Damon include all random measurement uncertainties and conservatively include any minor systematic bias associated with the standards or any other sources of error, then the calculated values in Table 6 should be correct.

In comparing the values in Table 6 with those in Table 1, it is unknown how many of the values in Table 1 from Damon were calculated. The values in Table 1 that differ significantly from the values in Table 6 appear to be incorrect. The largest difference occurs for the uncertainty for the weighted mean for Tucson. The calculated uncertainty in Table 6 is ± 17.18 but the value listed in Table 1 (Table 2 in Damon) is ± 31. It could not be determined how this ± 31 value was calculated. Industrial Chemist Remi Van Haelst (Ref. 8 to 11) has said that in discussions with the British Museum, they could not tell him how the one sigma uncertainty for the weighted mean for Tucson was calculated to be ± 31 in Damon when he also calculated ± 17. Thus, this ± 31 value appears to be an artificially assigned value instead of a value calculated from the measurement values and their one sigma uncertainties. The result of this increase in the uncertainty is to mask the underlying inconsistency between the laboratory's weighted means and the measurement uncertainties. In other words, to a large extent it hides the fact that there may be a significant systematic bias to the measured values that is causing a much larger spread in the laboratory weighted means than is justified by the measurement uncertainties. This decrease in the uncertainty for the weighted mean for Tucson from the apparent assigned value (± 31) in Table 1 to the calculated value (± 17.18) in Table 6 propagates through the calculation to decrease the final weighted mean and its uncertainty from 689 ± 16 in Table 1 to 672.46 ± 12.68 in Table 6. The effect of this is to increase the χ^2 value from 6.4 in Table 1 to 8.55 in Table 6, which decreases the significance level from 5% in Table 1 (which should actually be 4.18% as in Table 3) to 1.39% in Table 6. Thus, when the statistical analysis calculation is done consistently and correctly without any arbitrarily assigned values to increase the measurement uncertainty, the probability of the laboratory means being consistent with the specified measurement uncertainties is reduced to only 1.4%. This is based on the measurement uncertainties specified in Damon being composed of only random measurement errors. A probability of 1.4% that the spread in the measured values are consistent with the random measurement errors alone implies that there is about a 98% probability that a systematic bias is also present. If a systematic bias is present, it could be affecting all the measured values, and it would not be obvious how much the bias changed the measured values from the true value. Thus, the measured values in Damon should not be trusted as necessarily accurate.

Those doing the statistical analysis in Damon assumed that the low significance level of 5% in Table 1 was to be explained by the measurement uncertainties being understated. But this explanation becomes much less likely when the significance level is reduced from 5% in Table 1 to 1.4% in Table 6. The possibility that the correct significance level of 1.4% is caused by the measurement uncertainties being understated is investigated in Figure 4. The red dashed line in Figure 4 plots how the significance level increases as the uncertainty of Tucson's measurements is increased. This plot shows that if all the one sigma uncertainty values for all of Tucson's measurements is increased by 50%, the significance level only increases from 1.4% to 3.5%. It would take all of Tucson's uncertainty values to be increased by a factor of 3.0 to increase the significance level to 10%. The blue dashed line in Figure 4 shows that the uncertainties for all measurements at the Oxford and Zurich laboratories would have to be increased by 50% to increase the significance level to 10%. The black solid line in Figure 4 shows that the uncertainties for all measurements at all three laboratories would have to be increased by about 38% to increase the significance level to 10%. To prove that the inconsistency in the measurement data ought to be explained by an understatement in the uncertainties rather than by the presence of a systematic bias would require the significance level to be at least 50%. Figure 4 shows that it is not possible to reach a 50% level if only the Tucson measurement uncertainties are increased, but that a 50% level can be reached if the measurement uncertainties from the Oxford and Zurich laboratories are increased by a factor of 3.0 or if the measurement uncertainties of all three laboratories are increased by a factor of 2.5. None of these scenarios is credible since the stated uncertainty for each measurement is based on measurements of standards using the same equipment and procedures as the measurement of the samples.

A second reason that the measurement uncertainties for material 1 (Shroud samples) are not understated is that all three laboratories in Table 6 are in reasonable agreement with each other. A third reason that the uncertainties are not understated for material 1 is that they are in reasonable agreement with the uncertainties obtained for materials 2, 3, and 4 by all three laboratories. For material 1 (the Shroud samples), the average measurement uncertainty in Table 6 is 49.5 ± 6.0. This is within one sigma of the average measurement uncertainties of materials 2, 3, and 4 (45.8 ± 8.1, 45.6 ± 5.9, and 51.4 ± 18.5 respectively). If the measurement uncertainties are significantly underestimated, then it may be justified to also reject the measured dates for the samples since the measurements of the samples and the standards were most likely done using the same equipment and procedures. For all these reasons, an understatement of the measurement uncertainties must be rejected as causing the 1.4% significance level. The only other option is the presence of a systematic affect or bias, such as something changing the measured value based on the prior position of the sample on the Shroud.

6. Analysis of the Bias

To test for the possibility of a position dependent systematic bias in the C^{14} date measurements, the final weighted means for the three laboratories were plotted in Figure 3 as a function of the distance from the end of the Shroud. The y-values (C^{14} date, AD) for this plot were calculated from the weighted laboratory means in Table 6 using the formula: Date (AD) = 1950 – Date (YBP) with the date YBP (years before present) from Table 6 equal to 749.17 ± 30.70 for Oxford, 676.14 ± 23.74 for Zurich, and 646.52 ± 17.18 for Tucson. This resulted in a date (AD) of 1200.83 ± 30.70 for Oxford, 1273.86 ± 23.74 for Zurich, and 1303.48 ± 17.18 for Tucson. The x-value (distance from the bottom of the Shroud, cm) that should be used in Figure 3 can be obtained in at least two different ways:

1. The first option is based on the approximate measured values for the widths. These values are 1.4 cm for the samples sent to Oxford and Zurich, and 1.0 cm for sample A1 sent to Tucson. This makes a total width for these three samples of 3.8 cm. This option was not used to plot the data in Figure 3.
2. The second option is based on the measured weights of the samples. Though there has been some confusion in the literature as to the correct weights, the best values appear to be those listed in Table 8. When the Shroud was sampled, a piece 8.1 cm long was cut from the Shroud. It is believed that this piece was then cut approximately in half with about 4.3 cm on the left and 3.8 cm on the right (page 158 of Ref. 16). The right three samples (O, Z, and A1 in Figure 2) were cut from this right half so that the left side of the sample sent to Oxford was 4.3 cm from the bottom edge of the Shroud. The sample A2 was cut from the left half piece, i.e. the 4.3 cm length, so that when both A1 and A2 were sent to Tucson, they would receive close to the same total weight of sample as Oxford and Zurich. Table 8 shows how the widths of the samples were determined for the plot in Figure 3: 1.4192 cm for Oxford, 1.4410 cm for Zurich, and 1.0808 cm for Tucson's sample A1. The measured weights for the four samples in the third column of Table 8 were divided by an areal density of 0.0229 g/cm^2 (Ref. 17) to obtain the area of the sample in the fifth column. This area for the sample was then divided by the approximate height of 1.6 cm that is generally accepted for the samples to obtain the calculated width. It is recognized that the areal density of 0.0229 g/cm^2 and the specified height of 1.6 cm must be regarded as approximate values and so introduce an uncertainty into the sample widths. Nevertheless, the exact sample widths as listed in Table 8 were used in Figure 3 to not introduce additional round off error. With the left side of the Oxford sample at 4.3 cm from the bottom of the Shroud, the resulting center point of the three samples (O, Z, and A1 in Figure 2) that were C^{14} dated are as follows: x = 5.0096 cm for Oxford, x = 6.4397 cm for Zurich, and x = 7.7006 cm for Tucson's sample A1. These x-values, together with the above y-values for the date (AD), were used to plot the three measured values in Figure 3.

The red diamonds in Figure 3 are the weighted mean values calculated for the C^{14} dates, as discussed above. As shown in Figure 2, the sample labeled "O" was sent to the Oxford laboratory, was closest to the bottom of the cloth, and gave the oldest date (~ 1201 AD). The sample labeled "Z" was sent to the Zurich laboratory, was the middle sample, and gave the middle date (~ 1274 AD). And the sample labeled "A1" that was sent to the laboratory in

Tucson, Arizona, was furthest from the bottom of the cloth, i.e. closest to the center of the body mass, and gave the youngest date (~ 1303 AD). In Figure 3, the vertical red lines extending above and below each of the C^{14} dates is the measurement uncertainty (one standard deviation = 68% probability). When the scientists did the statistical analysis of this data, as reported in Damon, it was assumed that the C^{14} dates from the three laboratories could simply be averaged. In the technical literature, this assumption can be phrased in different ways:

- It was assumed that there is no contamination on the different samples sent to the three laboratories that would not be removed in the cleaning process, so that the samples were representative of the rest of the Shroud.
- It was assumed that the C^{14} isotopic ratio was uniform, or homogeneous, across the sample area and the Shroud.
- It was assumed that the samples were homogeneous, not heterogeneous, with the rest of the Shroud.
- It was assumed that only a random measurement error applied to the various C^{14} dates. These measurement uncertainties were only due to inherent uncertainties in the measurement equipment and procedures, and not due to any inherent differences in the samples.
- It was assumed that there was no bias, or systematic error component, between the different samples sent to the three laboratories, so that the only uncertainty was due to random measurement errors.

This assumption is represented in Figure 3 by the horizontal wide-dashed black line at 1260 AD, which is the average uncorrected (not corrected for the changing C^{14} concentration in the atmosphere) C^{14} date from the three laboratories as determined in Damon. It was this 1260 AD uncorrected value, or the resulting corrected range of 1260 to 1390 AD, that the media reported as proving that the Shroud was a forgery. But notice that this horizontal line at 1260 AD only goes through the uncertainty band of one point – the 1274 ± 24 value from Zurich. It misses the C^{14} dates from Oxford and Tucson. This indicates that simply averaging of the C^{14} dates does not appear to be justified, and that something might be causing a slope (also called a gradient) in the C^{14} values as a function of the distance from the end of the Shroud.

The optimum (best-fit) straight line (Y = A x + B where Y is the year AD) through these three points was determined by a weighted "least squares" method so that the uncertainty in the dates for the three points could be accounted for. In this method, the coefficients A and B are determined that minimize the weighted sum (S) of the squares of the deviations between the best-fit line and the plotted data points:

$$S = [(((A\,x_1 + B) - y_1)^2)/\sigma_1^2 + (((A\,x_2 + B) - y_2)^2)/\sigma_2^2 + (((A\,x_3 + B) - y_3)^2)/\sigma_3^2] \,/\, D$$

where the denominator $D = (1/\sigma_1)^2 + (1/\sigma_2)^2 + (1/\sigma_3)^2$
so that the weighting factor is the inverse of the square of the uncertainty = $(1/\sigma_i)^2$.

For simplicity, this equation was solved for “A” and “B” by an iterative process, resulting in the best fit equation Y = 35.87 x + 1030.67, which is the short-dashed red line plotted in Figure 3. This best fit line has a gradient or slope of 35.87 years per cm. This means that if the location from which the sample is cut from the Shroud is moved one cm further from the bottom edge of the Shroud, the C^{14} date would be expected to increase by about 36 years. And if the sample location is moved one inch (2.54 cm) further from the bottom edge of the Shroud, the C^{14} date would be expected to increase by about 91 years. And if the sample location is moved 13.5 inches (34.2 cm) further from the bottom edge of the Shroud and thus closer to the center of the body mass, the C^{14} date would be expected to increase by about 1230 years, assuming the extrapolation is valid. This is the difference between the time of Jesus and the lower limit obtained from the C^{14} dating (1260 AD).

If it is assumed that the Shroud was folded under the feet so that the radiocarbon sample area was directly below the feet on the centerline of the body when the neutrons were emitted, then the slope of the C^{14} dates would be about 57 years/cm, as presented at the Shroud conference in St. Louis (slide 21 of Ref. 18). To obtain a slope across the C^{14} dates of 35.9 years/cm as in Figure 3, the area of the Shroud from which the radiocarbon samples were removed must have been near the feet, but a small distance from under the body centerline. It has not yet been calculated how far the sample area must have been from under the body centerline, but it is known, based on the many MCNP calculations that were not reported in Ref. 18, that the slope decreases as the sample area is moved vertically up from the limestone bench or perpendicular away from the body centerline. The main point is that the best-fit slope through the three laboratory’s C^{14} dates of 35.9 years per cm, as in Figure 3, would be produced by neutrons emitted from within the body if the Shroud were wrapped so that the radiocarbon sample area was near the feet but not located directly under the feet on the centerline of the body. The area from which the Shroud was sampled could also have been above the feet. Thus, depending on how the Shroud was wrapped around the feet, the slope of 35.9 years/cm in Figure 3 can easily be produced by neutrons emitted from within the body.

The equation (Y = 35.87 x + 1030.67) for the best fit line in Figure 3 when solved for the three x-values for the sample locations (x = 5.0096, 6.4397, and 7.7006 cm from the bottom of the Shroud for the Oxford, Zurich, and Tucson A1 samples) give the Y-values Y = 1210.36, 1261.66, and 1306.89 AD, which is equivalent to 739.64, 688.34, and 643.11 YBP. These values can be used to test the effectiveness of the best fit line in Figure 3 to solve the inconsistency between the laboratory mean values and the specified measurement uncertainties that caused a significance level (probability of consistency) of only 1.4% in Table 6. This is accomplished by again using a χ^2 (chi squared) statistical analysis, as shown in Table 7.

As a reference, the first column of numbers in Table 7 is the same as the first column of numbers in Table 6, i.e. both are for material 1 which is the Shroud samples. The weighted mean of the weighted means, which is often called the grand or final mean, in this column is 672.46 ± 12.68. The second column of numbers in Table 7 is based on the deviation of the measurements from this final weighted mean of 672.46. Notice that the χ^2 and the significance level in the second column of numbers is the same as in the first column (χ^2 = 8.55, significance level = 1.39%). This means that the χ^2 statistical analysis can use either the measured values, as in the first column of numbers, or the deviation from the final weighted mean, as in the second column of numbers. Either way the same result is obtained. The third column of numbers is a test case. This column is based on the deviation of the measurements from the weighted mean for each laboratory in the first column of numbers, i.e. 749.17, 676.14, and 646.52 YBP for Oxford, Zurich, and Tucson, respectively. This test was successful because the results turned out as they should, with $\chi^2 = 0.0$ and the significance level = 100%, which means that there is a 100% probability that the laboratory weighted means are consistent with the measurement deviation from those laboratory weighted means. This is as it should be and was only included to show that the process in Table 7 is being done correctly.

The fourth column of numbers is the case of interest. This column considers the measurement deviations from the best fit line in Figure 3, i.e. from 739.64, 688.34, and 643.11 YBP for Oxford, Zurich, and Tucson, respectively. The result of the statistical analysis is $\chi^2 = 0.40$ which results in a significance level = 79.4%. This means that there is a 79.4% probability of the laboratory weighted means being consistent with the measurement uncertainties if it is recognized that there is something affecting the measurements that is a function of the distance from the bottom of the Shroud as given by the best fit line in Figure 3. If it is not recognized that there is something affecting the measurements as a function of distance from the bottom of the Shroud then the black dashed line (constant value = 1260 AD) in Figure 3 should be used, which produces only a 1.4% probability that the laboratory weighted means are consistent with the measurement uncertainties, as indicated in the first column of numbers in Table 7. In other words, if this systematic bias that is affecting the measurements is not recognized, then there is only a 1.4% probability of consistency between the laboratory mean values and the measurement uncertainties. In this case, the probability of consistency is so low that all measurements for the Shroud should not be accepted as necessarily accurate, and it ought to be recognized that there is something that is affecting the measurements as a function of the distance from the bottom of the Shroud. In statistical analysis terminology, this is called a spatially dependent systematic bias that is affecting the measurements.

According to the neutron absorption hypothesis discussed in Ref. 3, this is caused by the natural distribution of neutrons in the tomb that were included in the burst of radiation emitted from within the body that burned the image onto the Shroud. Some of these neutrons would have been absorbed in the trace amounts of N^{14} in the Shroud, which would cause new C^{14} atoms to be produced on the Shroud by the (N^{14} + neutron $\rightarrow$ C^{14} + proton) reaction. This is shown in Figure 5 by the vertical lines going upward at the zero-time point. These vertical lines indicate that the three samples that were dated by the three laboratories had absorbed a different number of neutrons because of the slope of the neutron distribution across the sample area, as shown by the curves at the second point from the left in Figures 9 to 11 in Ref. 3. This is shown in Figure 5 by the different vertical lines having different heights. The C^{14} content in the sample sent to the laboratory in Oxford must have increased by an average of 15.20% due to the neutrons absorbed in N^{14} producing new C^{14} in the Shroud. The C^{14} content in the sample sent to Zurich must have increased by an average of 16.24%. And the C^{14} content in sample A1 sent to Tucson must have increased by an average of 16.66%. These are the average amounts that the C^{14} must have increased in order produce the ages that were reported by Damon. The C^{14} in each of these samples, after being increased by neutron absorption at time zero in Figure 5, would have decayed with a half-life of 5730 years, as shown in Figure 5. The horizontal dashed lines and the vertical dashed lines in Figure 5 indicate how these decay curves would then be projected back onto the black curve that was assumed to be correct in Damon, thus producing the C^{14} dates indicated in Table 6.

In Figure 3, the best fit curve through the C^{14} date measurements was determined to be $Y = 35.87\,x + 1030.67$. The gives the slope in the "x" direction of about 36 years per cm, where x = the horizontal distance from the end of the Shroud in Figure 2. The MCNP computer code can also calculate the slope in the "y" direction, which is the vertical distance from the mid-height of the sample, when the MCNP input is set up properly to tally for this as an output item. This will be done in a future series of MCNP computer calculations. For simplicity at this point, it will be assumed that the x and y-components in the equation can be separated, and that the slope in the y-direction is the same as the slope in the x-direction. With these assumptions, the measured value for any point on the samples can be estimated as:

$$M_i = A + B(x_i, y_i) \pm R_i = 35.87\,(x - y) + 1030.67 \pm R_i$$

where M_i = C^{14} date AD that would be measured for sample i. This is the apparent C^{14} date for the sample.

A = Actual date for the sample based on the original C^{14} taken into the sample while the plant was alive, not including any C^{14} added to the sample due to neutron absorption in the cloth. This does not have an i subscript because it would be the same for all samples. The date for the manufacturing of the cloth is believed to be about 30 AD, so that A = 30 AD.

$B(x_i, y_i)$ = Bias for the sample from location (x_i, y_i). This is the change in the C^{14} date due to absorption of neutrons at location (x_i, y_i) producing new C^{14} atoms at that location. The "x" value is the horizontal distance in cm from the bottom of the Shroud in Figure 3 and the "y" value is the vertical distance from the mid-point of the sample in the y-direction.

R_i = One sigma distribution in the C^{14} date due to random variations in the measurements.

Since A = 30 in the above equation, the equation for the bias can be estimated as:

$$B(x, y) = 35.87\,(x - y) + 1000.67$$

In Figure 3, the samples extend from x = 4.3 cm to 8.241 cm based on the sample widths in Table 8: 1.4192 cm for Oxford, 1.4410 cm for Zurich, and 1.0808 cm for Tucson A1. The height of the samples is listed as 1.6 cm in Table 8, so the limits for the y-dimension will be from -0.8 to 0.8 cm. The above equation yields the values of the bias B(x, y) in Table 10 for various locations on the three samples that were C^{14} dated. This bias is the number of years that a date will be shifted forward when the C^{14} dating methodology is used. This bias is caused by the spatially dependent neutron absorption in the Shroud that results when neutrons are emitted from within the body. This distribution is based on the best fit equation in Figure 3 rather than on MCNP calculations because MCNP has not yet been run with sufficient resolution to produce the details of this distribution, though MCNP calculations have shown that a similar distribution can be produced depending on how the Shroud was folded at the instant that the burst of radiation occurred.

The values of the bias in Table 10 are used in Table 11 to produce the date for each location on the three samples in terms of the years before present (YBP). In Tables 10 and 11, the distributions are shown in a 7x8 array for the Oxford and Zurich samples, and is shown in a 5x8 array for the Tucson A1 sample. To obtain the correct width for each sample, the size of rectangles (Δx by Δy) in these tables are different for the three samples: 0.2027 x 0.2 cm for Oxford, 0.2059 x 0.2 for Zurich, and 0.2162 x 0.2 for Tucson A1. Table 12 compares the range of the measured values for the C^{14} dates from Table 6 with the values in Table 11 that were obtained as discussed above. The conclusion to be drawn from Table 12 is that there is reasonable agreement between the measured values and the Table 11 values given the magnitude of the measurement uncertainty for each measurement. Refinements in future MCNP calculations will allow the values in Tables 10 and 11 to be updated so that hopefully a better agreement with the measured values may be obtained.

7. Agreement of Previous Researchers with this Statistical Analysis

Other Shroud researchers have come to many of the same conclusions as stated above, sometimes as the result of very detailed statistical analysis of the C^{14} dating results:

- Paragraph 23 of Damon (Ref. 1): "More quantitatively, to establish whether the scatter among the three laboratory means (average values) was consistent with their quoted errors, a χ^2 test was applied to the dates for each sample … The results of this test … show that it is unlikely that the (measurement) errors quoted by the laboratories for sample 1 (samples from the Shroud of Turin) fully reflect the overall scatter." In other words, the stated measurement errors are not sufficient to explain the spread in the laboratory mean values for the C^{14} dates. This should have raised the possibility that there could have been be a systematic bias affecting the measured values. The items in parentheses in the above quote are added explanations for the reader.
- Industrial Chemist Remi Van Haelst has written several papers on the statistical analysis of the C^{14} dating of the Shroud (Ref. 8 to 11) in which he confirms the values listed in Table 6. Regarding Remi's statistical analysis, Professor Bene of the University of Geneva said "I would like to congratulate you for the quality of your work. You established definitive evidence, that the measurements made on the linen of the Shroud are not homogeneous and that they should be rejected."
- Statistician Bryan J. Walsh (page 7 of Ref. 12) said: "A re-analysis of the data indicates it is possible that the location of the sample was directly related to the radiocarbon measurement observed … the nature and cause of any proposed radiocarbon enhancement mechanism in linen fiber must be ascertained through a series of rigorously-controlled experiments …" According to the neutron absorption hypothesis (Ref. 3), this "radiocarbon enhancement mechanism" that Walsh referred to is due to neutron absorption in the Shroud.
- Bryan J. Walsh (page 5 of Ref. 12) said: "A regression analysis was then conducted which compared the subsample radiocarbon dates with the corresponding distance from the edge of the Shroud linen. It was determined that there was (a) statistically significant

(P > 98.8%, $r^2 = 0.49$) inverse linear relationship between the date (YBP) measured and the distance from the sample to the edge of the cloth. This finding indicated that there was an apparent gradient (i.e. slope) of radiocarbon measured on the Shroud sample with higher levels of C^{14} measured at an increasing distance from the edge of the Shroud linen based on the sample(s) measured." According to the neutron absorption hypothesis, the "higher levels of C^{14} measured at an increasing distance from the edge of the Shroud", as Walsh referred to it, is due to the natural distribution of neutrons in the tomb when they are released from within the body in a burst of radiation. Walsh's plot of all 12 measured C^{14} dates gave a gradient (slope) of about 42 years per cm. This gradient in the C^{14} dates is consistent with the nuclear analysis computer calculations of the neutron distribution within the tomb to be discussed in Ref. 3.

- Bryan J. Walsh in Ref. 13 used various statistical analysis techniques to analyze the C^{14} measurement data. These techniques consistently indicated that the samples sent to the three laboratories were not homogeneous, i.e. that they did not contain the same ratio of C^{14} to the other carbon isotopes. These techniques included:

    1. In a Kruskal-Wallis analysis of variance of errors only, a probability of statistically different C^{14} to carbon ratios = 97.5% was obtained for the three samples.
    2. In a Kruskal-Wallis analysis of variance of C^{14} dates only, a probability of statistically different C^{14} to carbon ratios = 95.1% was obtained for the three samples.
    3. A randomized analysis of variance gave a probability of statistically different C^{14} to carbon ratios = 97.7%.
    4. A Bonferoni pairwise T-test comparison indicated that the Oxford results were statistically different from the Arizona results at the 95% confidence level.

  This means that the C^{14} dates measured by the three laboratories were statistically different from each other, and not just different due to random measurement errors. Something must have caused the samples that were sent to the three laboratories to be inherently different in C^{14} content. Walsh concluded in the abstract that "Further analysis revealed that the sample dates observed were directly related to the physical location of the sample on the Shroud linen." But recognized in his last sentence that "The reason for this is not readily apparent". The neutron absorption hypothesis in Ref. 3 proposes an explanation that is consistent with everything that we know about carbon dating as it relates to the Shroud.

- Emanuela Marinelli (page 28 of Ref. 19): "It must be considered as likely the presence, in the analyzed piece of cloth (the Shroud of Turin), of an environmental contamination, which has acted in a non-uniform, but linear way, adding a systematic effect that is not negligible". According to the neutron absorption hypothesis, what is termed an "environmental contamination" in this quote is due to absorption of neutrons in N^{14} in the Shroud causing new C^{14} atoms to be produced in the Shroud.

- Riani, Atkinson, Fanti, and Crosilla (page 14 and 18 of Ref. 14): "…there is evidence of a trend in the age of the sample with the value of x_1." This means that the age/date depends on the distance (x_1) of the sample from the end of the Shroud. As explained above, this is due to the shape of the neutron distribution across the sample area, as will be discussed in Ref. 3. The quote continues "The presence of this trend explains the difference in the means that was detected by Damon et al. … The effect is that of a decrease in the age BP (Before Present where "present" = 1950) as x_1 (the distance from the end of the Shroud) increases. The effect is not large over the sampled region; between x_1 = 43 and 81 (mm from the end of the Shroud), our estimate of the change is about two centuries. Extrapolation of this linear trend to unsampled values of x_1 eventually leads to meaningless negative values." The authors are talking about the age becoming a negative value if the sample is taken far enough away from the end of the Shroud, but it is certainly not "meaningless", because a negative age is simply a date into the future, consistent with the calculations in Ref. 3. The quote continues: "Our explanation is that of greater contamination towards the center of the cloth." The authors conclude that there is "evidence of a strong linear trend" in the age/date as a function of the distance from the bottom of the Shroud. According to the neutron absorption hypothesis (Ref. 3), what these authors are calling "contamination" causing "a strong linear trend" is due to neutrons emitted from within the body, which caused a neutron distribution within the tomb (Figure 9 in Ref. 3), which caused a distribution of neutrons absorbed in N^{14} in the Shroud (Figure 10 in Ref. 3), which caused a distribution of new C^{14} atoms which caused a distribution in the C^{14} date (Figure 11 in Ref. 3). The items in parentheses in the above quotes are added explanations for the reader.

- Fanti and Malfi (page 155 to 161 of Ref. 16): "Generally the limit of significance level equal to 0.05 is used as a convention. If, like in the case under examination, a significance level obtained that is lower (than) this level of 0.05 (i.e. 1.39% in Table 6) … the existence of a not negligible bias … has to be hypothesized." (p.155) "Environmental bias that could have altered the results (of C^{14} dating) even of thousands of years has not been taken into consideration." (p.160) The authors recognize that this "bias" could shift the C^{14} date thousands of years. Consistent with this, Figures 11, 13, and 14 in Ref. 3 show that the predicted dates can be shifted by thousands of years, even into the future, based on the normal equations being used to calculate the dates from the C^{14} measurements. "The deviation of the obtained result can also be caused by an environmental effect linked to the body image formation. The body image formation process is not yet clear, but it would seem related to an intense burst of energy." (p. 161) This is consistent with the neutron absorption hypothesis, where the neutrons are included in a burst of radiation emitted from within the body that burns the image onto the Shroud. The "environmental effect" in this quote, according to the neutron absorption hypothesis, is the distribution that neutrons naturally take in the tomb when they are emitted from within the body.

The above indicates that Shroud researchers doing statistical analysis research on the C^{14} dating of the Shroud have come to the same conclusions as presented in this paper, with one exception. The new concept in this document is that the measured values were systematically altered depending on their prior position on the Shroud based upon neutron absorption in the N^{14} in the Shroud. It is most reasonable to believe that the neutrons were released in the burst of radiation that was emitted from within the body that burned the image onto the Shroud.

8. Conclusion

As indicated in Damon, et al. (Ref. 1), those doing the statistical analysis of the 1988 C^{14} dating of the Shroud of Turin recognized the inconsistency between the measured dates and the measurement uncertainties. To resolve this inconsistency, it was assumed in Damon, without evidence, that the measurement uncertainties were underpredicted. But this is highly unlikely because: 1) the uncertainties were most likely determined using the same equipment and procedures as the measurements, 2) the stated uncertainties are consistent between the various laboratories and the various standards, and 3) the magnitude that the uncertainties would have to be increased to eliminate the inconsistency is not credible. The only other alternative is that the variation in the measurements was not only caused by normal random measurement errors but also by a systematic bias. This is consistent with the laboratories in Tucson and Oxford measuring statistically different values for the date of the Shroud. And a Chi-squared statistical analysis of the 1988 C^{14} date measurements indicates only about a 1.4% probability that the variation in the measurements was caused only by random measurement errors. The only other option, that a systematic bias was also present, must therefore have a probability of about 98%. Plotting of the average values from the three laboratories yields a slope of about 36 years per cm measured from the bottom of the Shroud. This indicates that the systematic bias is the result of the original positions of the samples on the Shroud. Thus, each measurement [M_i] was being altered by a spatially dependent systematic bias [$B(x_i, y_i)$] so that:

$M_i = A + B(x_i, y_i) \pm R_i$, where

- M_i = C^{14} date that would be measured for sample i. This is the apparent C^{14} date for the sample.
- A = Actual age for the sample based on the original C^{14} taken into the sample while the plant was alive. This does not have an i subscript because it should be the same for a sample from any location.
- $B(x_i, y_i)$ = Bias for the sample from location (x_i, y_i). This is the change in the C^{14} date due to absorption of neutrons at location (x_i, y_i) producing new C^{14} atoms at that location.
- R_i = One sigma distribution in the C^{14} date due to random variations in the measurements.

This paper argues that the C^{14} date measurements were done correctly but the measurement data was misinterpreted because the presence of the bias term [$B(x_i, y_i)$] was not recognized. Multiple experts that have done statistical analysis on the 1988 measurement data concur with this conclusion (Ref. 8 to 15). The scientists who reported the statistical analysis in 1989 (Ref. 1) apparently did not seriously consider the possibility that a systematic bias could have affected the measurements, and thus their analysis did not include the depth of analysis that would have been necessary to discover such a systematic bias. Since the effect of this bias on the measurements was not recognized or quantified so that the measured values could be corrected for the bias, the C^{14} date of 1260 to 1390 AD should not be accepted as necessarily valid. Thus, the conclusion in Damon that "The results provide conclusive evidence that the linen of the Shroud of Turin is mediaeval" is not justified.

The main objection to the concept that there was a systematic bias that affected the measurements is the question of what caused the bias. The evidence indicates that the image on the Shroud was formed by a burst of radiation that was emitted from within the body (Ref. 21 and 22). The hypothesis that neutrons were included in this burst of radiation is the only concept that explains all four things that are known about C^{14} dating as it relates to the Shroud (Ref. 3). If neutrons were emitted from within the body, they would naturally take a cosine distribution within the limestone tomb (Figures 9 to 11 of Ref. 3). The neutrons would be absorbed in the trace amount of N^{14} on the Shroud which would produce new C^{14} on the Shroud by the (N^{14} + neutron → C^{14} + proton) reaction. The natural distribution of neutrons in the tomb would cause a position dependent systematic bias in the measurements that could shift the C^{14} date by thousands of years. To shift the C^{14} date at the sample location from about 30 AD to 1260 AD only requires the C^{14} concentration to be increased by about 16%. Such a burst of radiation is never emitted by a normal human body, whether alive or dead. In all our historical records, the only person and event that are suggested to explain a dead crucified body emitting such a burst of radiation is Jesus in his reported disappearance from within the tomb (John 20:3-9).

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Table 1. Measurements and Analysis Listed in Damon (Ref. 1)

	Material 1	Material 2	Material 3	Material 4
Source of material:	Shroud of Turin	Linen from tomb at Qasr Ibrim, Egypt	Mummy of Cleopatra from Thebes, Egypt	Cope of St. Louis d'Anjou of France
Expected date:		11th to 12th Century AD	110 BC to 75 AD	1290 to 1310 AD
Laboratory	**Individual Measurements* of C^{14} Date, Years Before Present (YBP, Present = 1950)**			
Tucson, Arizona	591 ± 30	922 ± 48	1838 ± 47	724 ± 42
	690 ± 35	986 ± 56	2041 ± 43	778 ± 88
	606 ± 41	829 ± 50	1960 ± 55	764 ± 45
	701 ± 33	996 ± 38	1983 ± 37	602 ± 38
		894 ± 37	2137 ± 46	825 ± 44
Zurich, Switzerland	733 ± 61	890 ± 59	1984 ± 50	739 ± 63
	722 ± 56	1036 ± 63	1886 ± 48	676 ± 60
	635 ± 57	923 ± 47	1954 ± 50	760 ± 66
	639 ± 45	980 ± 50		646 ± 49
	679 ± 51	904 ± 46		660 ± 46
Oxford, England	795 ± 65	980 ± 55	1955 ± 70	785 ± 50
	730 ± 45	915 ± 55	1975 ± 55	710 ± 40
	745 ± 55	925 ± 45	1990 ± 50	790 ± 45
Laboratory	**Mean C^{14} Dates (YBP) Based on Above Values**			
Tucson, Arizona	646 ± 31	927 ± 32	1995 ± 46	722 ± 43
Zurich, Switzerland	676 ± 24	941 ± 23	1940 ± 30	685 ± 34
Oxford, England	750 ± 30	940 ± 30	1980 ± 35	755 ± 30
	Analysis of Interlaboratory Scatter			
Unweighted mean of the weighted means **	**691 ± 31**	936 ± 5	1972 ± 16	721 ± 20
Weighted mean of the weighted means ***	689 ± 16	**937 ± 16**	**1964 ± 20**	**724 ± 20**
χ^2 value (2 degrees of freedom)	6.4	0.1	1.3	2.4
Significance level (%), ****	5	90	50	30

* - The values are from Tables 1 and 2 of Ref. 1. Uncertainties are 1 standard deviation.
** - Standard errors based on scatter. Bold values are final values in Ref. 1.
*** - Standard errors based on combined quoted errors.
**** - The probability of obtaining, by chance, a scatter among the three dates as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 2. Dates AD for the C^{14} Dating of the Shroud in 1988*

	Material 1, Shroud of Turin
Laboratory	**Individual Measurements***
Tucson, Arizona	1359 ± 30
	1260 ± 35
	1344 ± 41
	1249 ± 33
Zurich, Switzerland	1217 ± 61
	1228 ± 56
	1315 ± 57
	1311 ± 45
	1271 ± 51
Oxford, England	1155 ± 65
	1220 ± 45
	1205 ± 55
Laboratory	**Mean Values**
Tucson	1304 ± 31
Zurich	1274 ± 24
Oxford	1200 ± 30
	Analysis
Unweighted mean of the weighted means **	**1259 ± 31**
Weighted mean of the weighted means ***	1261 ± 16
χ^2 value (2 degrees of freedom)	6.4
Significance level (%), ****	5

* - Values calculated from the previous table based on Years Before Present = 1950. Uncertainties are one standard deviation.

** - Standard errors based on scatter. Bold values are final values used in Ref. 1.

*** - Standard errors based on combined quoted errors.

**** - The probability of obtaining, by chance, a scatter among the three dates as high as that observed, assuming that the quoted errors reflect all sources of variation.

Table 3. Recalculation of Last Four Lines from Laboratory Means

	Material 1	**Material 2**	**Material 3**	**Material 4**
Source of material:	Shroud of Turin	Linen from tomb at Qasr Ibrim, Egypt	Mummy of Cleopatra from Thebes, Egypt	Cope of St. Louis d'Anjou of France
Laboratory	**Mean C^{14} Dates* (YBP) Listed in Damon (Ref. 1)**			
Tucson, Arizona	646 ± 31	927 ± 32	1995 ± 46	722 ± 43
Zurich, Switzerland	676 ± 24	941 ± 23	1940 ± 30	685 ± 34
Oxford, England	750 ± 30	940 ± 30	1980 ± 35	755 ± 30
	Analysis of Interlaboratory Scatter			
Unweighted mean of the weighted means **	690.67 ± 30.91	936.00 ± 4.51	1971.67 ± 16.42	720.67 ± 20.22
Weighted mean of the weighted means ***	**689.12 ± 16.04**	**937.28 ± 15.86**	**1964.44 ± 20.41**	**723.85 ± 19.93**
χ^2 value relative to the above weighted mean (2 degrees of freedom)	6.35	0.138	1.303	2.386
Significance level (%), ****	4.18	93.4	52.1	30.3

* - Uncertainties are 1 standard deviation. ** - Standard errors based on scatter.
*** - Standard errors based on combined quoted errors.
**** - The probability of obtaining, by chance, a scatter among the three dates as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 4. Tucson's Eight Measurements of Shroud Samples

Date, 1988	**Measured Value (YBP)**	**Weighted Mean (YBP)**	**Weighted Sigma (YBP)**	**Date (YBP) Reported in Damon (Ref. 1)**
May 6	606 ± 41	591.49	30.31	591 ± 30
May 6	574 ± 45			
May 12	753 ± 51	690.08	35.33	690 ± 35
May 12	632 ± 49			
May 24	676 ± 59	605.66	40.99	606 ± 41
May 24	540 ± 57			
June 2	701 ± 47	701.00	33.23	701 ± 33
June 2	701 ± 47			

Table 5. Recalculated Statistical Analysis with 4 Tucson Values

	Material 1	Material 2	Material 3	Material 4
Source of material:	Shroud of Turin	Linen from tomb at Qasr Ibrim, Egypt	Mummy of Cleopatra from Thebes, Egypt	Cope of St. Louis d'Anjou of France
Expected date:		11^{th} to 12^{th} Century AD	110 BC to 75 AD	1290 to 1310 AD
Laboratory	**Individual Measurements of C^{14} Date, Years Before Present (YBP, Present = 1950)**			
Tucson, Arizona	591 ± 30	922 ± 48	1838 ± 47	724 ± 42
	690 ± 35	986 ± 56	2041 ± 43	778 ± 88
	606 ± 41	829 ± 50	1960 ± 55	764 ± 45
	701 ± 33	996 ± 38	1983 ± 37	602 ± 38
		894 ± 37	2137 ± 46	825 ± 44
Zurich, Switzerland	733 ± 61	890 ± 59	1984 ± 50	739 ± 63
	722 ± 56	1036 ± 63	1886 ± 48	676 ± 60
	635 ± 57	923 ± 47	1954 ± 50	760 ± 66
	639 ± 45	980 ± 50		646 ± 49
	679 ± 51	904 ± 46		660 ± 46
Oxford, England	795 ± 65	980 ± 55	1955 ± 70	785 ± 50
	730 ± 45	915 ± 55	1975 ± 55	710 ± 40
	745 ± 55	925 ± 45	1990 ± 50	790 ± 45
Laboratory	**Weighted Mean C^{14} Dates (YBP) Based on Above Values**			
Tucson, Arizona	646.44 ± 17.05	927.44 ± 19.70	1995.23 ± 19.89	721.67 ± 20.42
Zurich, Switzerland	676.14 ± 23.74	940.60 ± 23.16	1939.81 ± 28.47	685.16 ± 24.63
Oxford, England	749.17 ± 30.70	937.88 ± 29.43	1977.05 ± 32.71	755.76 ± 25.66
	Analysis of Interlaboratory Scatter			
Unweighted mean of the unweighted means (YBP)	695.09 ± 32.37	937.33 ± 6.26	1968.82 ± 14.74	732.16 ± 19.17
Unweighted mean of the weighted means (YBP)	690.59 ± 30.52	935.30 ± 4.01	1970.70 ± 16.31	720.86 ± 20.39
Weighted mean of the weighted means (YBP)	**672.21 ± 12.62**	**933.98 ± 13.37**	**1977.05 ± 14.59**	**720.16 ± 13.40**
χ^2 for weighted mean (2 degrees of freedom)	8.60	0.210	2.55	3.95
Significance level* (%)	1.40	90.1	28.0	13.9

* - The probability of obtaining, by chance, a scatter among the 3 laboratory weighted means as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 6. Recalculated Statistical Analysis with 8 Tucson Values

	Material 1	**Material 2**	**Material 3**	**Material 4**
Source of material:	Shroud of Turin	Linen from tomb at Qasr Ibrim, Egypt	Mummy of Cleopatra from Thebes, Egypt	Cope of St. Louis d'Anjou of France
Expected date:		11^{th} to 12^{th} Century AD	110 BC to 75 AD	1290 to 1310 AD
Laboratory	**Individual Measurements of C^{14} Date, Years Before Present (YBP, Present = 1950)**			
Tucson, Arizona	606 ± 41	922 ± 48	1838 ± 47	724 ± 42
	574 ± 45	986 ± 56	2041 ± 43	778 ± 88
	753 ± 51	829 ± 50	1960 ± 55	764 ± 45
	632 ± 49	996 ± 38	1983 ± 37	602 ± 38
	676 ± 59	894 ± 37	2137 ± 46	825 ± 44
	540 ± 57			
	701 ± 47			
	701 ± 47			
Zurich, Switzerland	733 ± 61	890 ± 59	1984 ± 50	739 ± 63
	722 ± 56	1036 ± 63	1886 ± 48	676 ± 60
	635 ± 57	923 ± 47	1954 ± 50	760 ± 66
	639 ± 45	980 ± 50		646 ± 49
	679 ± 51	904 ± 46		660 ± 46
Oxford, England	795 ± 65	980 ± 55	1955 ± 70	785 ± 50
	730 ± 45	915 ± 55	1975 ± 55	710 ± 40
	745 ± 55	925 ± 45	1990 ± 50	790 ± 45
Laboratory	**Weighted Mean C^{14} Dates (YBP) Based on Above Values**			
Tucson, Arizona	646.52 ± 17.18	927.44 ± 19.70	1995.23 ± 19.89	721.67 ± 20.42
Zurich, Switzerland	676.14 ± 23.74	940.60 ± 23.16	1939.81 ± 28.47	685.16 ± 24.63
Oxford, England	749.17 ± 30.70	937.88 ± 29.43	1977.05 ± 32.71	755.76 ± 25.66
	Analysis of Interlaboratory Scatter			
Unweighted mean of unweighted means (YBP)	695.38 ± 32.15	937.33 ± 6.26	1968.82 ± 14.74	732.16 ± 19.17
Unweighted mean of weighted means (YBP)	690.61 ± 30.51	935.30 ± 4.01	1970.70 ± 16.31	720.86 ± 20.39
Weighted mean of the weighted means (YBP)	**672.46 ± 12.68**	**933.98 ± 13.37**	**1977.05 ± 14.59**	**720.16 ± 13.40**
χ^2 for weighted mean (2 degrees of freedom)	8.55	0.210	2.55	3.95
Significance level* (%)	1.39	90.1	28.0	13.9

* - The probability of obtaining, by chance, a scatter among the 3 laboratory weighted means as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 7. Effect of a Systematic Bias on the Significance Level

	Measured C^{14} Dates (YBP)	Deviation from final weighted mean	Deviation from weighted means for each lab	Deviation from the best fit line in Figure 3
Laboratory	**Measurements**	**Deviation of the measured values**		
Tucson, Arizona	606 ± 41	-66.45 ± 41	-40.52 ± 41	-37.11 ± 41
	574 ± 45	-98.45 ± 45	-72.52 ± 45	-69.11 ± 45
	753 ± 51	80.56 ± 51	106.49 ± 51	109.89 ± 51
	632 ± 49	-40.45 ± 49	-14.52 ± 49	-11.11 ± 49
	676 ± 59	3.56 ± 59	29.49 ± 59	32.89± 59
	540 ± 57	-132.45 ± 57	-106.52 ± 57	-103.11 ± 57
	701 ± 47	28.56 ± 47	54.49 ± 47	57.89 ± 47
	701 ± 47	28.56 ± 47	54.49 ± 47	57.89 ± 47
Zurich, Switzerland	733 ± 61	60.56 ± 61	56.86 ± 61	44.66 ± 61
	722 ± 56	49.56 ± 56	45.86 ± 56	33.66 ± 56
	635 ± 57	-37.45 ± 57	-41.14 ± 57	-53.34 ± 57
	639 ± 45	-33.45 ± 45	-37.14 ± 45	-49.34 ± 45
	679 ± 51	6.56 ± 51	2.86 ± 51	-9.34 ± 51
Oxford, England	795 ± 65	122.56 ± 65	45.83 ± 65	55.36 ± 65
	730 ± 45	57.56 ± 45	-19.17 ± 45	-9.64 ± 45
	745 ± 55	72.56 ± 55	-4.17 ± 55	5.36 ± 55
Laboratory	**Weighted Means**	**Deviation of the laboratory means from the final weighted mean**		
Tucson, Arizona	646.52 ± 17.18	-25.93 ± 17.18	0.00 ± 17.18	3.41 ± 17.18
Zurich, Switzerland	676.14 ± 23.74	3.70 ± 23.74	0.00 ± 23.74	-12.20 ± 23.74
Oxford, England	749.17 ± 30.70	76.73 ± 30.70	0.00 ± 30.70	9.54 ± 30.70
Weighted mean of the weighted means (YBP)	672.46 ± 12.68	0.01 ± 12.68	0.00 ± 12.68	0.01 ± 12.68
	Analysis of Interlaboratory Scatter			
χ^2 for weighted mean (2 degrees of freedom)	8.55	8.55	0.00	0.40
Significance level* (%)	1.39	1.39	100.0	79.4

Uncertainties are 1 standard deviation.
* - The probability of obtaining, by chance, a scatter among the 3 laboratory weighted means as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 8. Description of C^{14} Samples from the Shroud

Designation in Figure 2	**Sent to Laboratory in:**	**Measured Weight (g)**	**Areal Density (g/cm²)**	**Calculated Area (cm²)**	**Approx. Height (cm)**	**Calculated Width (cm)**
A1	Tucson, Arizona	0.0396	0.0229	1.7293	1.6	1.0808
Z	Zurich, Switzerland	0.0528	0.0229	2.3057	1.6	1.4410
O	Oxford, England	0.0520	0.0229	2.2707	1.6	1.4192
A2	Tucson, Arizona	0.0141	0.0229	0.6157	1.6	0.3848

Measured weights from page 1 of Ref. 12 and pages 9 to 10 of Ref. 19
Areal density = 0.0229 g/cm^2 from page153 of Ref. 17
Approximate height of samples = 1.6 cm from page 1 of Ref. 20 and from
https://en.wikipedia.org/wiki/Radiocarbon_14_dating_of_the_Shroud_of_Turin

Table 9. Final C^{14} Date Without the Systematic Bias

	Original Calculation listed in Damon (Ref. 1)	Recalculated values in this paper
Data from:	Table 1	Table 6
Value in the Table	691 ± 31 YBP	672.46 ± 12.68 YBP
Rounded value (YBP)	690 ± 31 YBP	672 ± 13 YBP
Rounded value (AD). This is the uncorrected value.	1260 ± 31 AD	1278 ± 13 AD
Corrected range*	1260 to 1390 AD (95%)	1280 to 1300 AD and 1370 to 1380 AD (95%)

* - Corrected for changes in the C^{14} concentration in the atmosphere.

Table 10. Estimated Years the C^{14} Date would be Shifted due to Neutron Absorption

y-value	Oxford Sample							Zurich Sample							Tucson A1 Sample				
0.7	1133	1141	1148	1155	1163	1170	1177	1184	1192	1199	1207	1214	1221	1229	1236	1244	1252	1260	1267
0.5	1141	1148	1155	1162	1170	1177	1184	1192	1199	1206	1214	1221	1229	1236	1244	1251	1259	1267	1275
0.3	1148	1155	1162	1170	1177	1184	1191	1199	1206	1214	1221	1228	1236	1243	1251	1258	1266	1274	1282
0.1	1155	1162	1170	1177	1184	1191	1199	1206	1213	1221	1228	1236	1243	1250	1258	1266	1273	1281	1289
-0.1	1162	1169	1177	1184	1191	1198	1206	1213	1221	1228	1235	1243	1250	1258	1265	1273	1281	1288	1296
-0.3	1169	1177	1184	1191	1198	1206	1213	1220	1228	1235	1242	1250	1257	1265	1272	1280	1288	1296	1303
-0.5	1176	1184	1191	1198	1206	1213	1220	1227	1235	1242	1250	1257	1264	1272	1279	1287	1295	1303	1310
-0.7	1184	1191	1198	1205	1213	1220	1227	1235	1242	1249	1257	1264	1272	1279	1287	1294	1302	1310	1318
x =	4.40	4.60	4.81	5.01	5.21	5.42	5.62	5.82	6.03	6.23	6.44	6.65	6.85	7.06	7.27	7.49	7.70	7.92	8.14

Table 11. Estimated C^{14} Date, Years Before Present (YBP), Including above Bias B(x, y)

y-value	Oxford Sample							Zurich Sample							Tucson A1 Sample				
0.7	787	779	772	765	757	750	743	736	728	721	713	706	699	691	684	676	668	660	653
0.5	779	772	765	758	750	743	736	728	721	714	706	699	691	684	676	669	661	653	645
0.3	772	765	758	750	743	736	729	721	714	706	699	692	684	677	669	662	654	646	638
0.1	765	758	750	743	736	729	721	714	707	699	692	684	677	670	662	654	647	639	631
-0.1	758	751	743	736	729	722	714	707	699	692	685	677	670	662	655	647	639	632	624
-0.3	751	743	736	729	722	714	707	700	692	685	678	670	663	655	648	640	632	624	617
-0.5	744	736	729	722	714	707	700	693	685	678	670	663	656	648	641	633	625	617	610
-0.7	736	729	722	715	707	700	693	685	678	671	663	656	648	641	633	626	618	610	602
x =	4.40	4.60	4.81	5.01	5.21	5.42	5.62	5.82	6.03	6.23	6.44	6.65	6.85	7.06	7.27	7.49	7.70	7.92	8.14

Table 12. Range of Dates, YBP

Laboratory	Measured Values from Table 6	Values in Table 11
Tucson A1	540 to 701	602 to 684
Zurich	635 to 733	641 to 736
Oxford	730 to 795	693 to 787

Figure 1. Decay of C^{14} After Death

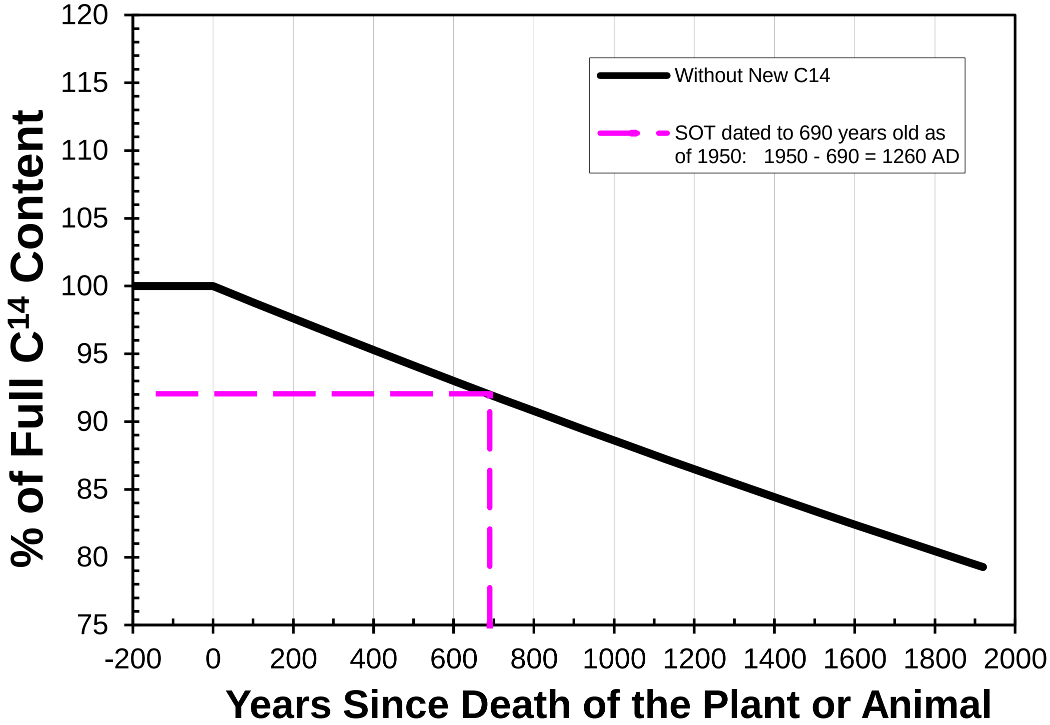

Figure 2. Location of Samples

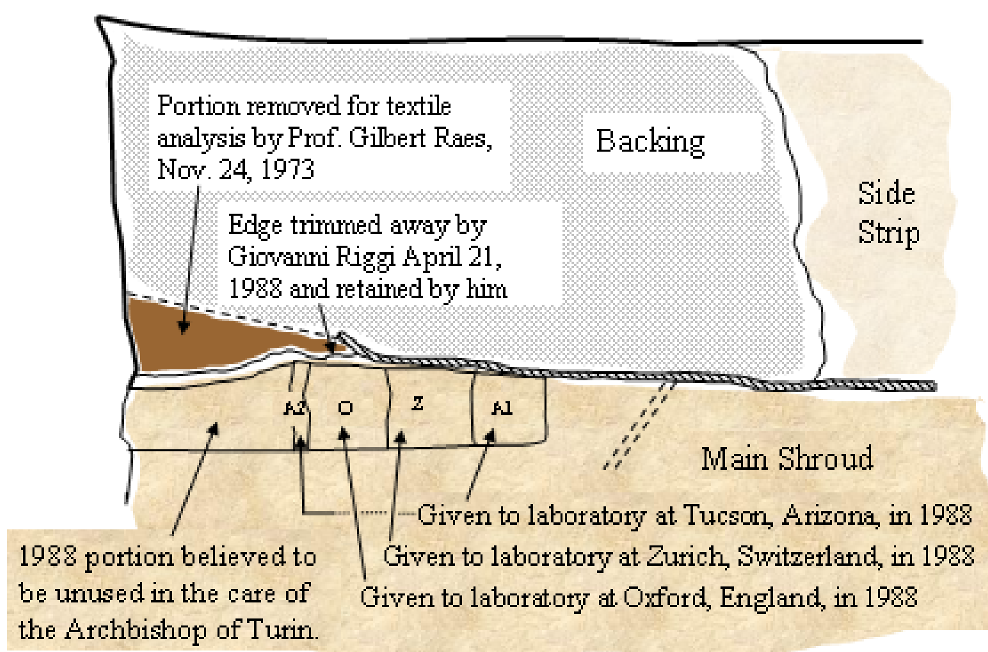

Figure 3. Average C^{14} Date from Each Laboratory,
Left to Right: Oxford, Zurich, and Tucson

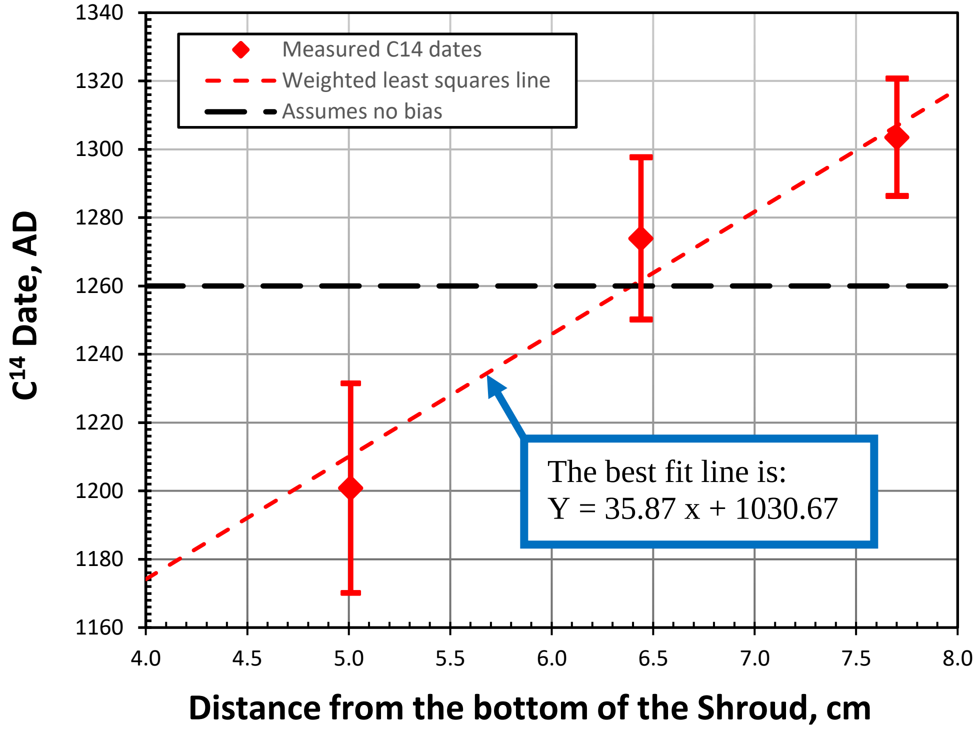

Figure 4. Significance Level for Larger Uncertainties

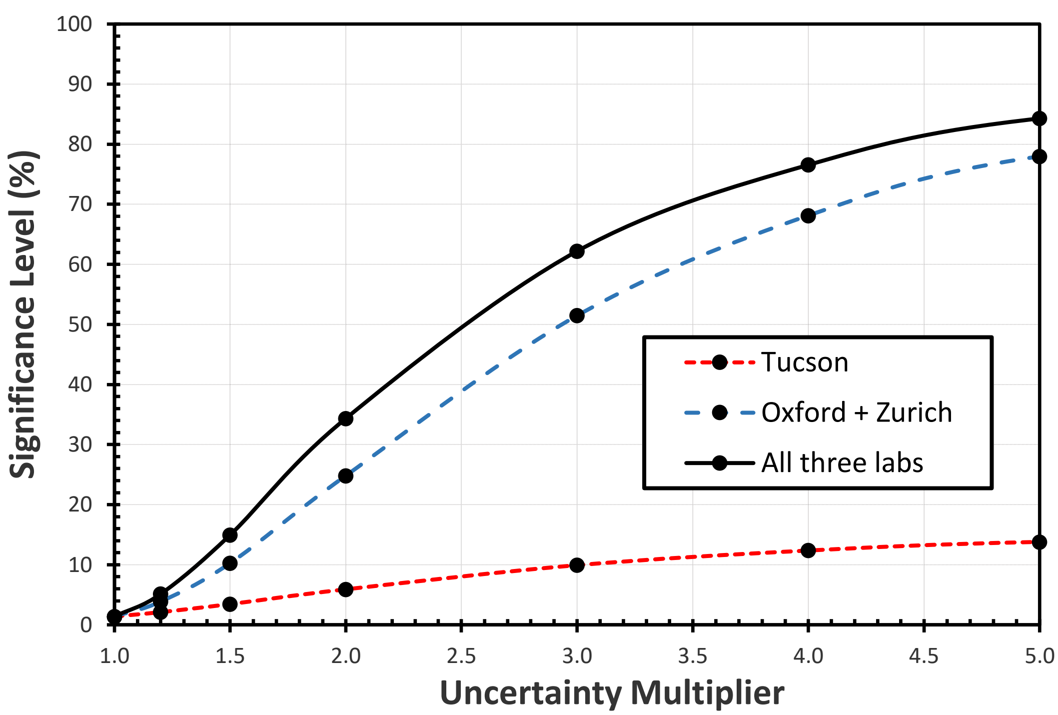

Figure 5. Different Increases in C^{14} for Each Sample Cause Different Dates