The Carbon Dating Problem for the Shroud of Turin, Part 1: Background

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Abstract

In 1988, the C^{14} dating methodology was used to date samples from the Shroud of Turin to 1260 to 1390 AD (Ref. 1). The problem with this date range is that it contradicts most other evidence which indicates that the Shroud of Turin is the authentic burial cloth of Jesus from the first century. To solve this carbon dating problem for the Shroud of Turin, a three-part series has been written that covers: 1) background, 2) statistical analysis, and 3) the neutron absorption hypothesis. This paper is part 1 in the series and covers background material that is needed to understand parts 2 and 3 in the series (Ref. 2 and 3). Basics of radiation and the C^{14} dating methodology are discussed to help people understand the neutron absorption hypothesis in Ref. 3. An extensive list of evidence is given why the C^{14} dating of the Shroud to the Middle Ages is not correct. A simplified example is given of measurements with and without a systematic bias, and the characteristics of the data analysis are explained that demonstrate when a systematic bias is present. This is required to understand the statistical analysis in Ref. 2.

1. Introduction

A burial cloth called the Shroud of Turin has been in Turin, Italy, since 1578. Ancient tradition claims that this burial cloth is the authentic burial cloth of Jesus of Nazareth. Amazingly, on the Shroud of Turin can be seen the front and back images of a naked man that was crucified exactly as the New Testament says that Jesus was crucified (Figure 1). According to the Biblical text (John 20:1-9), Jesus' empty burial cloth was discovered in the tomb on Sunday morning after Jesus' crucifixion. It is most reasonable to believe that the early Christians, due to persecution, would not have publicly revealed their possession of Jesus' burial shroud. They also would not have reused it, burned it, or thrown it out because it would have been extremely important to them because it had Jesus' blood on it and possibly his image, though the image may have appeared gradually due to aging and exposure to sunlight while it was being exhibited. If kept away from moisture, insects, fire, and intentional destruction, it should still be in existence due to the very slow rate of natural oxidation and dehydration of linen fibers. Given history over the last 2000 years, if Jesus' burial cloth were still in existence, the most likely place for it to be found is in association with the Roman Catholic Church. Thus, the cathedral in Turin, Italy, is a reasonable place for it to be located. Reliable historical records indicate that the burial shroud that is now in Turin came from Northern France where it was displayed in about 1355 or 1356 as the burial cloth of Jesus. Research over the last 50 years has revealed several lines of evidence (historical documents, traditions, works of art, images on coins, pollen, and DNA, Ref. 4 to 7) that indicate that prior to Northern France and Italy, it was probably in Constantinople, prior to that in Antioch in Syria, and prior to that in Jerusalem.

By the late 19^{th} century, these images on the Shroud of Turin were assumed to be paintings since there was a lack of evidence supporting the ancient tradition that it was Jesus' burial cloth. Scientific research on the Shroud during the 90-year period between 1898 and 1988 increasingly supported the authenticity of the Shroud, but the C^{14} dating of the Shroud in 1988 (Ref. 1, usually referred to as Damon) produced a date range of 1260 to 1390 AD, with a stated 95% probability that the true date is within this range. The conclusion of this C^{14} dating of the Shroud was that "The results provide conclusive evidence that the linen of the Shroud of Turin is medieval." (the abstract in Damon) But with 30 years of additional research, most Shroud researchers now believe that this C^{14} dating of the Shroud could not be correct since there are several lines of evidence that the Shroud existed long before 1260 AD. But it was also difficult to explain how the C^{14} dating could be so far off, dating the Shroud to the Middle Ages instead of to the first century. Thus, we have two lines of evidence, one, based on history and science, supporting the authenticity of the Shroud, and the other, based on C^{14} dating of the Shroud to the Middle Ages, strongly against authenticity.

How is this conundrum to be resolved? Resolution of these two lines of evidence (favoring and opposing authenticity) is accomplished through consideration of both sets of evidences, careful statistical analysis of the C^{14} measurement data, and detailed nuclear analysis computer calculations. Thus, whether the C^{14} dating of the Shroud samples to the Middle Ages disproves the authenticity of the Shroud cannot be properly judged without further statistical analysis of the C^{14} measurement data listed in Damon, et al. as well as consideration of the context of the larger scientific issues.

This paper is part 1 of a three-part series that attempts to solve the 1988 carbon dating problem for the Shroud of Turin. Part 2 in this series (Ref. 2) reports on a detailed statistical analysis of the 1988 measurement data, concluding that the measurements were very likely affected by a systematic bias so that the measurement results are not necessarily valid. And part 3 in this series (Ref. 3) proposes that the neutron absorption hypothesis explains why the C^{14} dating was affected by a systematic bias, which caused the Shroud of Turin to be dated to the Middle Ages instead of to about 30 AD.

This paper, as part 1 in the three-part series, lays the foundation for understanding parts 2 and 3, and should be understood by the layman before progressing to Ref. 2 and 3. This paper covers the nature of atoms and radiation (Section 2) so that the C^{14} dating methodology can be understood (Section 3). This background is needed to understand the neutron absorption hypothesis in Ref. 3. This paper then summarizes the history of scientific research on the Shroud (Section 4) leading up to the carbon dating in 1988, considers whether the C^{14} dating technique should necessarily be accurate for the Shroud (Section 5), and discusses the evidence that has convinced most Shroud experts that the Shroud is much older than the C^{14} date of the Middle Ages (Section 6). A statistical analysis of a simplified example of measurements with and without a systematic bias is then presented to demonstrate the effect of a systematic bias (Section 7). The material in Sections 4 to 7 is needed to understand and be receptive to the statistical analysis of the 1988 C^{14} measurement data in Ref. 2.

2. Understanding Atoms, Isotopes, Neutrons, and Radiation

Perhaps for some it would be helpful at this point to describe the nature of atoms, isotopes, neutrons, and radiation. The reader that is familiar with these topics can skip to the next section. Things like oxygen, carbon, hydrogen, nitrogen, calcium, phosphorus, etc., are called elements. These are the most common elements that compose the typical human body (with their weight percent): oxygen (65%), carbon (18%), hydrogen (10%), nitrogen (3%), calcium (1.4%) and phosphorus (1.1%). The smallest indivisible piece of every element is called an atom. Atoms of an element behave in the same way when they interact with atoms of other elements, which is what allows each element to be distinguished from other elements. To understand why this is so, it is necessary to understand what makes up an atom. Figure 2 shows a diagram of a carbon atom. An atom is similar in some respects to our solar system. The vast majority of the mass in our solar system is concentrated at the center of the solar system in the sun, and the much lighter planets move around the sun in their orbits. This is also true in an atom. The vast majority of the mass in an atom is concentrated in the nucleus at the center of the atom, and the much lighter electrons circle around the nucleus in their orbits. The nucleus is composed of protons and neutrons, which can be thought of as very small spheres of matter. Each proton has a positive (+1) electrical charge. Each neutron has zero electrical charge. In Figure 2, the protons are the red spheres in the nucleus and the neutrons are the blue spheres in the nucleus. Each electron has a negative (-1) charge. In every carbon atom, there are six protons in the nucleus and six electrons in their orbits around the nucleus. In any element, the number of electrons circling the nucleus is the same as the number of protons in the nucleus so that the atom is electrically neutral (zero net charge), with the positive electrical charges of the protons exactly balanced by the negative electrical charges of the electrons. The diagram in Figure 2 shows a carbon atom with its six protons in the nucleus and six electrons in their orbits around the nucleus. An atom of a different element would have a different number of protons in the nucleus with the same number of electrons circling the nucleus as the number of protons in the nucleus. For example, every oxygen atom has eight protons in the nucleus and eight electrons in their orbits circling the nucleus. In our solar system, each planet has its own orbit around the sun, but due to quantum mechanical effects in an atom, multiple electrons can circle the nucleus in the same orbit up to a specified maximum. As shown in Figure 2, each carbon atom has two electrons in an inner orbit, which fills this inner orbit. There are also four electrons in the outer orbit, though this outer orbit can contain up to eight electrons. For this reason, each carbon atom tries to fill the outer orbit by sharing electrons of other atoms, including other carbon atoms. This sharing of electrons binds the atoms together in what are called covalent bonds, thus allowing carbon to form millions of different organic compounds, which allows life to exist on the earth.

Though all atoms of an element contain the same number of protons and electrons, they can contain different numbers of neutrons. These are called isotopes of the element. The neutrons in the nucleus serve an important function. It is well known that like charges repel but opposite charges attract. This means that two positive charges, such as two protons (+1 each), would repel each other, and two negative charges, such as two electrons (-1 each), would also repel each other, but that opposite charges, such as an electron (-1) and a proton (+1), would attract each other. The question then arises as to why the multiple protons (+1 each) in a nucleus don't repel each other, thus tearing the nucleus apart. The answer is that it's the neutrons that hold the protons together in the nucleus. As shown in Figure 2, to prevent the six protons in the nucleus from repelling each other because like charges repel, which would destroy the nucleus, they must be held together by neutrons in the nucleus. 99% of all carbon atoms are the C^{12} isotope, with 6 protons and 6 neutrons in the nucleus, thus making a total of 12 total protons + neutrons in the nucleus. 1% of all carbon atoms are the C^{13} isotope, with 6 protons and 7 neutrons in the nucleus, thus making a total of 13 total protons + neutrons in the nucleus. Only a very small fraction of all the carbon atoms are C^{14} atoms, containing 6 protons and 8 neutrons. For most calculations, the atom fraction of C^{14} in carbon at the surface of the earth is usually assumed to be 1.0×10^{-12} (about one C^{14} atom per trillion carbon atoms) though it varies depending on the

C^{14} production rate in the upper atmosphere from cosmic rays, and by various terrestrial sources such as nuclear reactors and nuclear weapons testing. The ratio of neutrons to protons (8 / 6 = 1.33) in C^{14} atoms is evidently too high so that the C^{14} nucleus is not stable. As a result, C^{14} atoms decay with approximately a 5730-year half-life. This means that in 5730 years, only half of the initial number of C^{14} atoms would still exist, the rest having decayed. And in another 5730 years, the number of C^{14} atoms would be reduced by half again, thus leaving only 1/4th of the initial number of C^{14} atoms. A C^{14} atom decays by one of the neutrons emitting an electron thus changing into a proton ($C^{14} \rightarrow N^{14}$ + electron). This natural process of the decay of the C^{14} atoms in a material, such as the linen Shroud, is what allows the C^{14} dating methodology to work.

Every human body is made up of organs and tissue composed of cells, which are made up of organic molecules such as proteins, which are made up of atoms, which are composed of neutrons, protons, and electrons. Thus, every human body is made up of a very large number of neutrons, protons, and electrons. This indicates that neutrons are a natural part of the elements in the human body. Neutrons make up about 45% of the weight of an average human body. We don't normally encounter neutrons simply because they are bound up with the protons in the nuclei ("nuclei" is the plural of nucleus) of the various elements in the human body.

Radiation can be of two basic types – electromagnetic radiation and particle radiation. At a low energy, electromagnetic radiation has some characteristics of a wave in a medium, such as a wave on water. But at high energy, electromagnetic radiation has some characteristics of a particle. Having these apparently opposite characteristics at the same time is called the duality of electromagnetic radiation. The smallest unit of electromagnetic radiation is often called a photon, particularly at the high-energy range. A photon can be defined as a packet or bundle of electromagnetic energy. It has no mass and no electric charge but travels at the speed of light, which in a vacuum is about 300,000,000 meters per second (about 186,000 miles per second). The energy range of electromagnetic radiation covers a very wide spectrum that, from low energy to high energy, includes radio waves, micro waves, infra-red, visible light, ultraviolet, X-rays, and gamma rays. Photons of electromagnetic energy anywhere within this spectrum differ only by their energy. Thus, photons of ultraviolet light, X-rays, or gamma rays are basically the same type of things as photons of visible light except they have different energies. Experimental results indicate that electromagnetic radiation in the ultraviolet range can produce a straw-yellow discoloration of linen like that found in the image on the Shroud (Ref. 8 to 12).

Particle radiation, on the other hand, consists of very small rapidly moving bits of matter. The particle in this radiation would have weight, may have electrical charge, and would move very rapidly but slower than a photon. Typical particle radiation can consist of neutrons, protons, electrons, alpha particles (helium nucleus = 2 protons + 2 neutrons), or a host of more exotic particles (neutrinos, mesons, baryons, muons, etc.). In this paper, we are focused on the sub-atomic particles that make up the atoms that are normally in the human body: neutrons, protons, and electrons. Experimental results indicate that charged particles such as protons can also produce a straw-yellow discoloration of linen like that found in the image on the Shroud (Ref. 13 and 14).

3. Understanding the C^{14} Dating Methodology

The reader should also have a general understanding of how C^{14} dating is done. The Shroud is linen, which is made from long fibers from the stem of the flax plant. Carbon is a major component of these fibers. As explained above, all carbon atoms have six protons in the nucleus (the dense mass at the center of each atom) to electrically balance the six electrons that orbit around the nucleus in carbon atoms, but the number of neutrons in the nucleus can vary thus producing the various "isotopes" of carbon. The isotopes of carbon include C^{12}, C^{13}, and C^{14}, where the superscript refers to the total number of the protons plus neutrons in the nucleus. Since there are six protons in the nucleus of each carbon atom, then the C^{12}, C^{13}, and C^{14} isotopes have 6, 7, and 8 neutrons in each atom, respectively. The ratio of neutrons to protons (8/6 = 1.33) is too high in C^{14} atoms so they decay with a half-life of about 5730 years, which means that half of any specific number of C^{14} atoms will decay in this amount of time. The C^{12} and C^{13} atoms are stable, i.e. do not decay.

New C^{14} atoms are produced primarily in the upper atmosphere by cosmic rays from outer space but are also produced in nuclear reactors and nuclear weapons testing. The new C^{14} atoms gradually diffuse throughout the atmosphere until a small fraction is taken in by growing plants during photosynthesis. While the flax plants used to make the Shroud were growing, the C^{14} already in the plants was decaying but this loss of C^{14} atoms was exactly compensated by new C^{14} atoms being brought into the plant in the process of photosynthesis. This is shown in Figure 3 by the horizontal black line to the left of the zero age on the x-axis. The zero age in Figure 3 is assumed to be when the flax plant is cut down and made into the linen that went into the Shroud. The black line in Figure 3 shows that the amount of C^{14} in the Shroud decreases after it is cut down since after the death of the plant no new C^{14} is being brought into the flax fibers by photosynthesis. The magnitude of this decrease in the C^{14} content causes the 5730-year half-life of C^{14}. This allows the date of linen to be determined by measurement of the amount of the C^{14} isotope remaining in the linen in comparison to the C^{12} and C^{13} isotopes, with the assumption that the various carbon isotopes have not been added to or removed from the sample since it was cut down.

In 1988, when the scientists measured the ratio of C^{14} isotope to the C^{12} and C^{13} isotopes in the Shroud samples, they evidently measured about 92% of the C^{14} that would have been present when the flax plants were alive. Believing that the density of C^{14} atoms must be following the black decay curve in Figure 3 as time progresses, those doing the analysis of the C^{14} dating would use the black decay curve to conclude that the Shroud is about 690 years old (relative to 1950), as shown by the horizontal and vertical dashed lines in Figure 3. As a result, they assigned an uncorrected date (not corrected for changes in C^{14} concentration in the atmosphere) of 1950 – 690 = 1260 AD to the Shroud. The validity of this approach is discussed below.

4. History of Scientific Research on the Shroud Prior to 1988

In 1898, an amateur photographer named Secondo Pia took the first photograph of the face on the Shroud of Turin. The negative plate that he developed in the dark room turned out to contain a high resolution positive image of the face, which meant that the image on the cloth was a high resolution negative image. Contrary to the public's general belief, this implied that the image on the Shroud of Turin could not be a painting or other type of artistic creation because no artist up to that time could create a high-resolution negative image of a face since prior to the invention of photography one had ever seen a negative image of a face. Subsequent investigation of the Shroud in the first half of the 20th century involved mostly specialists in medicine, biology, and anatomy who concluded, based upon the exacting and pristine nature of the blood marks on the Shroud, that the Shroud must have wrapped the dead body of a crucified man and that this dead body in some unknown way must have produced the image and the blood marks that can be seen on the Shroud. Discoveries in the 1970s were consistent with these earlier conclusions:

- Investigation by researchers at the California Institute of Technology's Jet Propulsion Laboratory (JPL) using Fourier Transform analysis concluded that there were no brush strokes or any other evidence of directionality in the image of the face. This also proved that the image could not be a painting.
- In 1973, microscopic examination of the threads found that the discoloration that makes up the image is only located on the top one or two layers of fibers in a thread, and that there is no evidence of capillarity (soaking up of a liquid) of the discoloration. This proved that the discoloration could not be due to any liquid, such as a dye, stain, acid, or a chemical in solution.
- The Shroud was in a fire in 1532. Ray Rogers, then a chemist with the Los Alamos National Laboratory (LANL), realized that the temperature gradient that would have resulted from this fire would have caused changes in the discoloration if the image was due to either an inorganic or an organic chemical, but the discoloration showed no change due to the temperature gradient. Thus, the discoloration that makes up the image could not be due to application of either an inorganic or organic chemical. Ray Rogers also recognized that the temperature experienced by the Shroud in this fire was likely less than 100 degrees C in most places since the blood on the Shroud did not display the chemical characteristics of higher temperatures.
- Photographs and works of art such as paintings, being two-dimensional (2D) items, do not contain three-dimensional (3D) information content. But analysis of a photograph of the Shroud by a NASA VP-8 image analyzer indicated that the image on the Shroud contains 3D or topographical information content related to the vertical distance between the body and the cloth as the body was wrapped within the Shroud. This proved that the image on the Shroud is totally unique and could not have been produced by a photographic or painting process.
- The above discoveries led to the formation of the Shroud of Turin Research Project (STURP) and in 1978 resulted in about 24 scientists from leading research facilities in the United States going to Turin, Italy. They took about 2.5 million dollars' worth of scientific equipment with them. They were given five days, 24 hours per day, to do any tests that they wanted on the Shroud provided they didn't damage it. Their primary goal was to determine how the image was formed. They concluded that the image on the Shroud was not due to paint, stain, or dye. It was not a rubbing, a dusting, a print, a photograph, or a scorch from a hot object. It was also not due to a naturalistic process of chemicals that might have been put onto the body during the burial process interacting with body decay products, since no burial chemicals or body decay products were

detected on the Shroud. Thus, they concluded that they had no idea how the image was formed.

- Subsequent analysis by STURP of very small samples removed from the Shroud indicated that the discoloration on a fiber (there are about 100 to 200 fibers in a linen thread) was thinner than a wavelength of light, i.e. the straw-yellow discoloration was only about 0.2 microns (millionths of a meter) thick on a fiber, which has a diameter of about 20 microns. This discoloration layer extends 360 degrees around the circumference of the fiber, but the inside of the fiber is not discolored at all. The straw-yellow discoloration of this outer layer does not result from any material added to the fiber but results from some of the electrons in the outer orbit of the carbon atoms in the cellulose molecule being changed from single electron bonds to double electron bonds. But how could this be done to create an image of a naked crucified man on the linen Shroud?

Based on the above scientific evidence, by the mid-1980s most individuals that were fully aware of this research believed the following:

- The Shroud covered a real human body of an individual that had been crucified, and that this body in some unknown way produced the image and the blood marks that are on the Shroud.
- The technology required for an artist or forger to produce the unique characteristics of the image has not existed in any previous era, and does not even exist today, so that the image could not have been produced by any artist or forger.
- The characteristics of the image on the Shroud are so unusual that all attempts to find a naturalistic process to form the image would certainly fail.

By the mid-1980s, the weight of this scientific evidence caused some researchers to believe that the most likely cause of the image was radiation emitted from the body (Ref. 18 and 19) as it was wrapped within the Shroud. Based on this, an increasing fraction of the public believed that the Shroud of Turin was probably the authentic burial cloth of Jesus.

5. Should the C^{14} Dating Methodology Necessarily be Accurate for the Shroud?

By the mid-1980s, researchers were seriously considering the use of the C^{14} dating methodology to date the Shroud, but a unique issue arises at this point. Christian belief holds that Jesus' body disappeared from within the Shroud, leaving behind the collapsed burial shroud in the tomb (John 20:6-9). If the disappearance of Jesus' body from within the Shroud is an actual historical event, as Christians believe (Ref. 15), then it would be an event that is outside or beyond our current understanding of the laws of science. Several possibilities have been suggested as to how physically such a disappearance could have occurred, but the best option, in physics terms, appears to be that the disappearance of the body from within the Shroud was a transition into an alternate dimensionality (Ref. 16). The nature of such a transition is illustrated in Ref. 17. If this was the case, we would have no idea as to what other phenomena could accompany such an event. It might be possible for a burst of radiation to have been emitted in the process of the atoms in the body disappearing from our physical reality, so that radiation could have been emitted from within the volume of the body as it disappeared. And if this was the case, then neutrons might have been included in such a burst of radiation. The mere concept of the disappearance of the body from the tomb as described in John 20:6-9 neither requires nor prohibits a burst of radiation including neutrons from being emitted during the process of the body disappearing. It can only be said that if Jesus' body disappeared from within the Shroud, then it might be possible for neutrons to also have been emitted. The effects that would result from such an event need to be understood.

If neutrons were emitted from within the body, then a small fraction of them would have been absorbed in the trace amount of nitrogen that is typically in linen. It is well known that when a N^{14} atom absorbs a neutron, it produces a C^{14} atom and a proton by the [N^{14} + neutron $\rightarrow$ C^{14} + proton] reaction. The new C^{14} atoms that would be produced in the Shroud by this process would be indistinguishable from the original C^{14} atoms brought into the plant during growth of the plant, so that the newly produced C^{14} atoms would shift the apparent age of the linen in the positive direction, perhaps hundreds or even thousands of years forward. Thus, it should be realized that if the disappearance of Jesus' body from within the Shroud was a real historical event, as Christians believe, then there would be no basis to necessarily trust the dating of the Shroud by the C^{14} dating methodology. The process by which the body disappeared from the tomb might have caused the apparent C^{14} date to be shifted significantly forward. It would all depend on how many neutrons might have been emitted from the body and the sample location on the Shroud.

However, the individuals that owned and controlled the Shroud in the 1980s evidently did not realize that if the disappearance of Jesus' body from within the Shroud was a real historical event then the C^{14} date could have been shifted by hundreds or even thousands of years. As a result, the owners of the Shroud judged the C^{14} dating methodology to be so trustworthy that they not only allowed the C^{14} dating to be done, but also canceled an entire array of other important experiments until the results of the C^{14} dates were known.

6. Evidence that the Shroud was not made in the Middle Ages

Carbon dating of an object is based on measurement of the amount of C^{14} isotope in comparison to the C^{12} and C^{13} isotopes because C^{14} atoms decay with a half-life of about 5730 years whereas C^{12} and C^{13} atoms do not decay, i.e. are stable. In 1988, samples were cut from the lower-left corner of the Shroud when it is oriented vertically. These samples were sent to three laboratories in Tucson (Arizona), Zurich (Switzerland), and Oxford (England) for C^{14} dating. When the average values from these three laboratories were averaged, a value of 1260 ± 31 AD (1 sigma) was obtained. This is the uncorrected value. When this value was corrected for the changing C^{14} concentration in the lower atmosphere, a range of 1260 to 1390 AD (2 sigma) was obtained. The midpoint of this range is 1325 AD, which is sometimes quoted for the date of the Shroud. These are the values in the statistical analysis of the measured values as reported by Damon, et. al., in the British journal Nature in 1989 (Ref. 1). Based on these values, it was concluded that the Shroud of Turin could not be the authentic burial cloth of Jesus but instead was a forgery from the Middle Ages. The typical layman would have taken this conclusion as authoritative since it was based on the scientific methodology of C^{14} dating, appeared in the very reputable peerreviewed British journal Nature, and had 21 leadings scientists as authors. But scientists involved with C^{14} dating are very aware that contamination of a sample can cause the C^{14} dating methodology to produce very wrong results. And after 30 years of additional study on this issue by the Shroud research community, most researchers believe that this interpretation of the results from the C^{14} dating of the Shroud in 1988 was very flawed. The four categories of evidence are as follows:

- The impossibility of forming the image on the Shroud during the Middle Ages.
- The C^{14} dating of the Shroud in 1988 violated the internationally established protocols for C^{14} dating of the Shroud.
- Many evidences indicate that the Shroud is much older than the C^{14} date.
- Detailed statistical analysis of the C^{14} dating measurements indicate that the data is not consistent due to the very likely presence of a systematic bias that affected all the measurements. Unless the bias can be quantified to correct the measured values, the measurement data ought not be accepted as necessarily valid.

Summaries for the above four categories of evidence are discussed below.

6A. Impossibility of Creating the Image in the Middle Ages

A five-day hands-on investigation by the Shroud of Turin Research Project (STURP) in 1978 proved that the front and back images of the crucified man on the Shroud contain no pigment, no carrier, no brush strokes, no clumping of anything between the threads or the fibers, and no cracking of the image along the fold lines. Based on this, the images cannot be caused by paint, stain, or dye. STURP also proved that the threads and fibers display no capillarity (soaking up of a liquid) so that the images could not be due to a liquid such as an acid or an organic or inorganic chemical solution. And STURP proved that the images could not be due to a scorch from a hot object or a photographic process. Subsequent analysis by STURP proved that the straw-yellow discoloration that forms the image is only on the top one or two fiber layers in a thread, with the “top layers” of the thread defined as those facing the body. The discoloration on a fiber is 360 degrees around the outside circumference of the fiber with a discolored thickness of only about 0.2 microns into the 15 to 20-micron diameter of a fiber. The inside of the fiber not discolored. The discoloration on the outside 0.2 microns of the fiber is caused by a rearrangement of the electron bonding of the carbon atoms that were already in the cellulose molecules that make up the linen fibers. Thus, the discoloration is due to energy added to the cloth to change the way in which the four electrons in the outer orbit of the carbon atoms are shared with the surrounding atoms, but without material/atoms being added to the cloth. And the energy that was added to the cloth must have been added in a short burst so that the electron bonding could be altered before the energy was dissipated.

It is important to realize that this change in the electron bonding of the carbon atoms must be done in a pattern that creates the image of a naked crucified man. How could this be done? Energy to change the electron bonding, but without the addition of atoms, is a good description of radiation. Because of this and many other reasons (Ref. 18 and 19), it is concluded that the image is a radiation burn resulting from a burst of radiation that must have been emitted from within the body that was wrapped within the Shroud. Because of these image characteristics, there is no known process by which the image could have been made by an artist or forger in the Middle Ages. The technology to make these images did not exist in the Middle Ages and does not exist even today. Also, an artist or forger in the Middle Ages would not have known or been able to:

- Place serum rings (visible only under ultraviolet light) around the blood exudate of the scourge marks on the Shroud.
- Add pollen onto the Shroud that is unique to the Jerusalem area, or add pollen around the head that is from a plant with long thorns.
- Put a microscopic amount of dirt in abrasions on the tip of the nose and on one knee.
- Put bilirubin into the blood. Bilirubin is an organic chemical that is produced by the liver in the normal process of it cleaning up waste that arises from aged or damage red blood cells. An extremely high bilirubin concentration was measured in the blood on the Shroud. The flogging that Jesus is reported to have received (Figure 1) would have damaged a significant fraction of his red blood cells sufficient to produce a very high concentration of bilirubin in the blood.
- Place nails in the wrists rather that the palms and fold the thumbs under, contrary to paintings from the Middle Ages.
- Put microscopic chips of travertine aragonite limestone onto the Shroud containing impurities that closely match the limestone from a first century tomb in Jerusalem.
- Put nanoparticles of creatinine bound to ferritin onto fibers of the Shroud. These nanoparticles are typical of a person that had been heavily tortured (Ref. 23).
- Use a stitch that evidence indicates is from the first century to sew the three-inch wide side strip to the main Shroud.
- Create a negative image that contains 3D or topographical information content related to the body-to-cloth distance.

6B. Violation of Established Protocols

Multiple international conferences were held prior to 1988 to determine the required procedures to produce a trustworthy value for the C^{14} dating of the Shroud of Turin. In 1988, essentially all these protocols were violated in cutting the samples from the Shroud, measuring the C^{14} quantity in the samples, and doing and reporting the statistical analysis of the measured values. The most significant violation was that the samples sent to the three laboratories (Tucson, Zurich, and Oxford) came from only one location – the lower left corner of the Shroud when it is positioned vertically. Since C^{14} dating requires the burning of the sample, the samples sent to the three laboratories were initially all next to each other at the lower left corner of the Shroud, so only one location on the Shroud was sampled. Other locations on the Shroud would have produced very different C^{14} dates according to the neutron absorption hypothesis (Ref. 3). The procedures and problems related to the dating of the Shroud are summarized in Chapter 8 of Ref. 6, Chapter 14 of Ref. 7, and Ref. 29.

6C. Evidence the Shroud is Older than 1260 AD

There are 14 indicators that should be considered in dating the Shroud. 13 of these indicators are consistent with the time of Jesus. Only the C^{14} date is inconsistent with the time of Jesus. These dating techniques are listed below starting from the technique that gives the most recent date, and then proceeding back to older dates.

1. As discussed above, samples cut from the Shroud in 1988 were C^{14} dated at three laboratories, with the average of the laboratory average values being 1260 AD ± 31 years. This is the raw or "uncorrected" value. When this value was corrected for the changing concentration of C^{14} in the atmosphere, a range of 1260 to 1390 AD was obtained (Damon). This is a two-sigma range, which means that there should be a 95% probability that the true value is within this range. The one-sigma uncertainty outside of this 1260 to 1390 AD range is the same as for the uncorrected value = 31 years. Sometimes the midpoint of this range is quoted for the date of the Shroud, i.e. (1260 + 1390) / 2 = 1325 AD. Section 6D discusses why this conclusion results from an incomplete statistical analysis of the data.

2. The Hungarian Pray Codex or Manuscript is historically dated to 1192 to 1195 AD. It includes a painted drawing that must have been copied from the Shroud of Turin based on the pattern of burn holes on the painting and on the Shroud, so the Shroud must have existed in 1192 to 1195 AD. This is 65 years (1260 - 1195 = 65) prior to the range of the C^{14} date (1260 to 1390 AD, two sigma). Since one sigma for the C^{14} date is 31 years, 65 years prior to the C^{14} date range is an additional two-sigma (65 / 31 = 2.1), which means that the Shroud existing in 1192 to 1195 AD is four-sigma below the C^{14} date range. This is because the lower value of 1260 AD in the C^{14} date range is a two-sigma limit. Thus, the Shroud's existence in 1192 to 1195 AD, proven by this historical document, is four-sigma below the C^{14} date, which is far outside of the usual two-sigma acceptance criteria. And the burial cloth that was painted on the Hungarian Pray Manuscript had evidently been in Constantinople for centuries. This indicates that the C^{14} date range of 1260 to 1390 AD should be rejected. But more importantly, it raises the following question: What could have caused the C^{14} date to be so wrong? The best answer to this serious question is the neutron absorption hypothesis, which hypothesizes that the burst of radiation that caused the image included neutrons (Ref. 3). This neutron emission resulted in neutron capture in the Shroud, which created new C^{14} in the Shroud at the sample location primarily by the [N^{14} + neutron → C^{14} + proton] reaction. This must have caused about a 16% increase in the C^{14} content in the Shroud samples, which would have shifted the C^{14} date from the true value (30 to 33 AD) to the apparent value (1260 AD, uncorrected).

3. It is believed that the spinning wheel was invented in Asia by the 11th century and had spread to Europe by the 13th century. Since the Shroud is made of hand-spun thread, the threads that compose the Shroud were probably spun before the 12th century.

4. The international standard of the market place in the time of Jesus was the Assyrian cubit which was equal to about 21.6 inches (54.9 cm). The dimensions of the Shroud in

this unit is 7.97 by 2.02 cubits. When held up for display, the Shroud was normally held by the long side of the cloth with the lower side of the cloth hanging free. This would have caused the width to increase slightly during each such display, thus probably causing the length to decrease slightly. This means that the original dimensions of the Shroud were very likely 8 by 2 Assyrian cubits, consistent with the international standard used in the market place in Israel in the first century (Ref. 20). The cubit was evidently being used in some places even in the Middle Ages, but the use of the cubit in the manufacturing of the Shroud indicates that it was more likely made prior to the Middle Ages.

5. Ancient coins that contain the same image as the Shroud of Turin go back to about 675 AD. The Shroud of Turin and the coins could not have both been copied from another item since the Shroud could not have been copied from anything for reasons given above in Section 6A. This indicates that the coins must have been copied from the Shroud of Turin, thus showing that the Shroud must have existed prior to about 675 AD.

6. The face or head cloth of Jesus that Peter and John found in the Tomb on Sunday morning after Jesus' death and burial is believed to be in Oviedo, Spain. It is called the Sudarium of Oviedo, based on the Greek word (soudarion) in John 20:7. It does not contain an image. This is evidently because it was not on the face when the body disappeared from within the Shroud. But the Sudarium appears to contain the same type of blood as the Shroud of Turin and several researchers believe that the shape of the blood stains on the Sudarium match the locations on the head that were bleeding as indicated on the Shroud. Thus, there is good evidence that the Shroud of Turin and the Sudarium of Oviedo covered the same body. There is a definite history for the Sudarium that dates back to 570 AD in Jerusalem. It left Palestine in 614 and arrived in Spain a few years later. It went to northern Spain in 718 and was taken to Oviedo in 840 AD, where it has remained ever since. The evidence that the Sudarium and the Shroud covered the same body indicates that the Shroud can also be dated back to at least 570 AD.

7. Ancient paintings and other works of art that contain the same image as the Shroud of Turin go back to about 550 AD. For the reasons stated above, the ancient paintings must have been copied from the Shroud, so that the Shroud must have been in existence by about 550 AD.

8. Constantine the Great, the first Christian emperor, abolished crucifixion in the Roman Empire in 337 AD out of veneration for Jesus Christ, its most famous victim. Because the image on the Shroud is that of a crucified man, the image on the Shroud was probably made earlier than 337 AD, though crucifixion was occasionally used at later times and in other cultures.

9. Ancient historical documents and traditions indicate that the burial cloth of Jesus, after being in Jerusalem, was evidently in Antioch in Syria and Edessa in Turkey. This was probably in the first to third centuries. It may have also been in Galatia in Turkey (Gal. 3:1). Later, it was evidently in Constantinople tell after 1200 AD. It surfaced 150 years

later when it was publicly displayed in Lirey, France, about 1355 or 1356. It has been in Turin, Italy, since 1578.

10. There is a 3.5-inch wide piece of linen that is sewn onto the main piece of the Shroud along the entire left side of the Shroud. According to expert opinion, the stitch used to connect this side piece onto the main piece was made by a professional and is a unique stitch. The most similar stitch is on a piece of cloth found at Masada, which was destroyed in 73 to 74 AD. Thus, this stitch on the Shroud is probably one of the best ways to date the Shroud and dates it to the first century.

11. The image on the Shroud is that of a naked man who was crucified exactly as the Bible says that Jesus was crucified (Figure 1). As discussed above, many evidences indicate that the image could not be due to an artist or forger. For these and many other reasons, most Shroud researchers believe that the image was probably made in some way by the body that was wrapped in the Shroud, and that it probably was Jesus' body. Analysis of the ancient historical texts indicates that Jesus probably died either in 30 or 33 AD.

12. A photograph of the face on the Shroud taken by professional photographer Giuseppe Enrie in 1931 indicates a possible coin over one eye. With computer enhancement, three letters on the coin can be identified that indicate that it is a Roman Lepton minted by Pontius Pilate in 29 to 32 AD. This evidence is tentative, as it is found on only this one photograph and could be the result of the image enhancement. But with confirmation, this dating technique could become definitive.

13. Giulio Fanti performed three different types of physical tests to determine how flax fibers change with age (pages 204, 207, and 246 of Ref. 21). These tests were then applied to the Shroud to determine its age. The resulting ages are given below:

    - Fourier Transform Infrared Spectroscopy (FTIR): 300 BC ± 400 years
    - Raman Spectroscopy: 200 BC ± 500 years
    - Tensile strength of flax: 400 AD ± 400 years.

    The stated uncertainty values are two sigma values, equivalent to a 95% probability range. The average of the three tests is 33 BC ± 250 years for the Shroud of Turin.

14. Fibers from the Shroud show damage from sources of natural background radiation. Using microscopic analysis of the Shroud fibers, chemist Ray Rogers found that the radiation damage to the Shroud fibers indicates that the Shroud "is quite old, similar to flax fibers from the Dead Sea Scrolls" (page 5 of Ref. 22), which are dated to about 250 BC to AD 70. This indicates that the Shroud of Turin should also date to about the same period.

6D. Statistical Analysis of the C^{14} Measurements

The mean values reported by the laboratories in Tucson and Oxford were statistically different from each other at the 2.9 sigma level. Based on a chi-squared statistical analysis, the spread in all sixteen measurements has only a 1.4% probability (Table 6) that it was caused only by random measurement errors. This implies there is a very significant probability (98.6%) that something other than random measurement error, i.e. a systematic bias, was also affecting the C^{14} date measurements. Previous statistical analysis of the 1988 C^{14} date measurements (Ref. 24 to 28) also concluded that there was a systematic bias present. Since this systematic bias was not quantified and corrected for, the C^{14} dates should not be accepted as necessarily valid. When plotted as a function of the distance from the bottom of the Shroud, the C^{14} dates reported by the three laboratories have a slope of about 36 years per cm (Figure 7), with the distance in cm measured from the bottom of the Shroud. This indicates that something changed the C^{14} concentration on the samples as a function of the original position of the samples on the Shroud. A detailed statistical analysis of the C^{14} date measurements is given in Ref. 2.

Therefore, the C^{14} dating of the Shroud to 1260 to 1390 AD should not be thought of as invalidating the authenticity of the Shroud. But the C^{14} date of 1260 to 1390 AD should not be thought of as merely incorrect. Instead, it should raise the following question: Most evidence indicates the Shroud is a first century piece of linen, so what could cause the C^{14} dating methodology to date it to the Middle Ages (1260 to 1390 AD)? When it is realized that this is the real question, the C^{14} dating of the Shroud to the Middle Ages becomes an important clue as to what happened to the Shroud in the first century. There is significant evidence that the image was formed by a burst of radiation from the body (Ref. 19). If this burst of radiation included neutrons, then a small fraction of these neutrons would have been absorbed in the trace amount of N^{14} on the Shroud so that new C^{14} would have been produced on the Shroud by the [N^{14} + neutron $\rightarrow C^{14}$ + proton] reaction. The C^{14} density on the samples would have to increase by only about 16% to shift the C^{14} date from the true value (30 to 33 AD) to the apparent value of 1260 AD. This is the neutron absorption hypothesis (Ref. 3).

7. Example of a Statistical Analysis of Measurements with a Systematic Bias

In doing any kind of measurement, the value obtained can be different than the true value. If a series of measurements are made of the same thing, this will cause a variation in the measurement results. The reasons for this can be divided into two causes: random measurement errors and systematic errors. Random measurement errors might cause the measured value to be a little high one time and a little low another time, but these errors are random, and the cause is not usually easily identified. Analysts don't usually care about the exact cause for random measurement errors because they can include the effect of these random errors in their mathematical analysis by assuming that the random measurement errors have a Gaussian distribution, which they usually do.

A Gaussian distribution, which is shown in Figure 4, is often called a normal distribution or a bell curve. A Gaussian distribution is the probability distribution of a measurement error that peaks at zero change and falls off symmetrically for either positive or negative changes, as in

Figure 4. 68.3% of the area under the Gaussian curve falls within what is called one sigma. 95.6% of the area under the curve falls within two sigma and 99.7% of the area under the curve falls within three sigma, as shown in Figure 4. These values are for a Gaussian distribution for many measurements.

In a series of measurements, the values can also be changed in a non-random, or "systematic" way by such things as changes in the equipment or samples, electrostatic fields, location, temperature, humidity, gravity, etc. These changes in the measured values are called systematic errors, can affect all the measurements, and can result in significant changes in the measured values. The effect of these systematic errors on the measurements is usually referred to as a systematic bias, so that the bias is the amount that each measurement is changed from the true value. The word "systematic" is used in contrast to the word "random".

Changes in the measured value due to normal variations in the measurement equipment are usually considered to be random variations in the sense that there is no pattern to them, so that a random variation can be simulated by the roll of dice. But a systematic bias can be expressed as an equation, i.e. as a function of some parameter, because a pattern can be discovered in the bias. Since random measurement errors and systematic bias are the only two options for explaining why repeated measurements of the same thing can vary, if it is proven that random measurement errors are very unlikely to be the only cause for the variation in the measurements, then the only other option is that a systematic bias is affecting the measurements. This is the case for the C^{14} dating of the Shroud that was done in 1988, as shown in Ref. 2.

To demonstrate a statistical analysis of measurements with and without a systematic bias, a simplified sequence of measurements will be considered. First, consider the case without a systematic bias. Assume a sealed container with a variety of materials in it including 60.000 milligrams (mg) of a certain element. A milligram (mg) is a thousandth of a gram. Also assume a device that can, without opening the container, measure the number of milligrams of this element in the container within a certain measurement uncertainty. Assume that the number of milligrams of this element in the container is measured on three successive days, with three measurements on day 1, five measurements on day 2, and eight measurements on day 3, for a total of 16 measurements. These number of measurements (3, 5, and 8) were chosen because the three laboratories that participated in the C^{14} dating of the Shroud in 1988 performed 3, 5 and 8 measurements. The random measurement error of the assumed device will be specified by the roll of two dice. This series of measurements is shown in Table 1. The first column is the measurement number in this sequence with the day given in the second column. The actual number of milligrams of this element in the container (60.000 mg) is in the third column, and the sum of the two dice when they were rolled is given in column 4.

A die has six faces numbered 1 through 6 so the average value obtained for one die is (1 + 2 + 3 +4 + 5 + 6)/6 = 3.5, so the average value for the sum of two dice is 7.0. There are 6 x 6 = 36 combinations for two dice, with 24 of these combinations adding to a sum between 5 and 9, inclusive. 24 out of 36 reduces to a ratio of 2/3, i.e. 66.7% of the possible values fall within the range of 5 to 9, inclusive. As noted above, in a Gaussian probability distribution for a large number of measurements, 68.3% of the values are expected to fall within a one sigma standard deviation of the mean value as shown in Figure 4. This value of 68.3% is very close to 66.7% of the time in which two dice will add to a value between 5 and 9, inclusive. Thus, for two dice, the one sigma variation about the average of 7.0 can be approximated as 2.0, i.e. (9 - 5)/2 = 2. Thus, 2.0 mg is specified for the random variation for each measured value to represent the one sigma measurement uncertainty. The sum of the two dice rolled in column 4 minus the average value (7.0) yields the deviation or delta from the average value. This delta is listed in column 5 of Table 1 and is the value used to simulate the random measurement error. The total measured value (column 6) is the sum of the actual value (column 3) plus the random measurement error in column 5.

The above example (Table 1) included only random measurement errors. The situation changes significantly when we add a systematic bias to the same 16 measurements in Table 1. Thus, in Table 2, we have the same number of milligrams of material in the container (60.000 mg) and the same random measurement error simulated by the same rolls of the two dice (column 4 of Table 2). But we now assume there is also a systematic bias (column 5 of Table 2) based on the number of total measurements that the measurement equipment has made. In this sequence of measurements, the measurement equipment is assumed to have made 29 prior measurements, so that the 16 measurements that are made on days 1, 2, and 3 are measurement numbers 30 to 45 as shown in Table 2 instead of measurements 1 to 16 in Table 1. Measurement 1 is assumed to have no bias, but each additional measurement is assumed to drift high by an additional 0.264 mg. For example, starting with new measurement equipment, without any random measurement error, the measurements would be 60.000, 60.264, 60.528, 60.792, and 61.056 mg, etc. In Table 2, the systematic bias (it's systematic because it's a function of the measurement number) goes from 29 x 0.264 = 7.656 mg for measurement 30 up to 44 x 0.264 = 11.616 mg for measurement 45. The bias value in column 5 of Table 2 must be added to the actual value in the container in column 3 (60.000 mg) plus the random measurement error in column 4 to produce the measured values in column 6 of Table 2. From these 16 measurement values can be calculated the weighted means for each day (67.253, 69.776, and 71.192 mg) and their uncertainties, as shown in Table 3. And from these values for the three days can be calculated the overall or final mean and its one sigma uncertainty (70.011 ± 0.500 mg), so that the two-sigma range would be from 70.011 – 2*0.500 = 69.011 mg up to 70.011 + 2*0.500 = 71.011 mg.

It might be tempting for the analysts to stop at this point and claim that the true value is between 69.011 and 71.011 mg with a 95% confidence, because two-sigma was used in the calculation of these boundaries. (It should be noted that this is what was done in the C^{14} dating of the Shroud of Turin in Damon.) But in this hypothetical case, to claim that the true value is between 69.011 and 71.011 mg with a 95% confidence is to make a significant error because the true value in this example is 60.000 mg, with the higher measured value (70.011 ± 0.500 mg) resulting from the systematic bias that was affecting all the measurements. It should be realized at this point that this hypothetical example was set up to illustrate why the C^{14} dating of the Shroud that was done in 1988 (Damon) obtained an uncorrected date of 1260 AD. In the hypothetical example, the measured value with the random measurement error and the systematic bias was 16.7% higher than the true value (70.011 / 60.000 = 1.167). 0.6% was due to the random measurement error (0.375 from Table 1 / 60.000 = 0.6%). And 16.1% was due to the systematic bias (9.636 from Table 2 / 60.000 = 16.1%). This 16.1% increase due to the systematic bias was used to set up the hypothetical example because if the C^{14} content in the samples cut from the Shroud in 1988 was increased by 16%, it would have caused the C^{14} date for the Shroud to be shifted from 30 AD to the apparent value of 1260 AD.

Thus, those doing the statistical analysis should not be satisfied with merely saying that the true value is between 69.011 and 71.011 mg with a 95% confidence. They should also look for the possibility that a systematic bias was affecting the data. A first step would be to plot the data to determine if there is a slope or gradient to the data as a function of various possible parameters. When the measurement data in Tables 1 and 2 is plotted as a function of the measurement number, Figure 5 is obtained. At the bottom of this figure, the 16 measurements without bias at about 60 mg shows a slight upward slope in the trendline calculated in EXCEL, but this is only due to random variations in the measurements simulated by the two dice. For the case with bias shown at the top of Figure 5, the 16 measurements at about 70 mg shows a significant upward slope in the trendline calculated in EXCEL that indicates the possible presence of a systematic bias in the measurements. Notice that if a systematic bias is detected, then it must be investigated and quantified if possible so that the measured values can be corrected. If the measured values are not corrected for the bias, then the measured values should not be accepted as necessarily valid because the systematic bias may be significantly affecting them. In the example being considered, the value moved from the true value (60.000 mg) to the measured value of 70.011 mg due to the bias. If the statistical analysis of the measured values indicates the presence of a systematic bias, the measurements should only be utilized if the magnitude of the bias can be determined so that the measured values can be corrected for the bias.

Another way to determine if a systematic bias is present over the three days of measurements is to plot the mean values for each day with their uncertainties, and then determine how well the average value and a trendline pass through the one sigma uncertainty bands. This is done in Figure 6 for the case with bias. Notice in Figure 6 that the final mean calculated for all 16 measurements (70.011 mg in Table 2), indicated by the horizontal black dashed line, only passes through the one sigma uncertainty band of one point but that the trendline, indicated by the sloped red dashed line, passes through the uncertainty bands of all three points. This indicates a probable bias affecting the measurements.

In Table 1, for the measurements without bias, the best estimate of the amount of the element in the container is the average value – the sum of the 16 measurements (966.00 mg) divided by 16, which equals 60.375 mg. This value is close to the true value assumed to be in the container in this example (60.000 mg) and is different only because of the random measurement errors simulated by the roll of two dice. An alternative method that gives the same answer (60.375 mg) is to calculate the average (called the mean in statistical analysis) for each day and the uncertainty for each day (59.333 ± 1.155, 60.800 ± 0.894, and 60.500 ± 0.707 mg), as given in the second column in Table 3. The uncertainty for each day is calculated from the square root of the sum of the inverse of the squares of each measurement uncertainty. When the weighted mean of the three-daily means is calculated, the answer is again 60.375 mg (Table 3). This is pointed out because it is used in later analysis.

The probability of whether all measurement variations are only due to random errors, or whether a systematic bias is probably present, can be numerically determined using a χ^2 (chi squared) statistical analysis. There are other methods that may be better, but this method was chosen because it was used in Damon. Thus, to determine whether the range or spread in the mean values for each day's measurements (59.333, 60.800, and 60.500 mg) is consistent with the random measurement error of 2.0 mg for each measurement, a χ^2 (chi squared) statistical analysis is used to calculate a significance level. This method is discussed in standard textbooks on statistical analysis. The important thing to realize is that this is an accepted methodology to determine whether the spread in the means is consistent with the random measurement error, and was the methodology used in Damon. If the spread in the means is not consistent with the random measurement error, then it ought to be determined whether the measurements were affected by a systematic bias. If they were, then to make use of the measured values, the systematic bias must be quantified and corrected for in the measured values. If the measured values are not corrected for the bias, then the values should not be accepted as necessarily valid. In the present case without bias, the calculated $\chi^2 = 1.0708$ so that the significance level = 58.54% as shown in Table 3. Values of the significance level listed in this paper are from http://www.socscistatistics.com/pvalues/chidistribution.aspx with two degrees of freedom. A significance level of 58.54% means that there is a 58.5% chance that the spread in the mean values from the three days is consistent with the random measurement errors, so that the measurement values ought to be accepted as valid.

The χ^2 statistical analysis for the case with bias is shown in the right column in Table 3. This analysis shows that $\chi^2 = 8.56$ which results in a significance level of 1.38%. This means that there is only a 1.4% chance that the spread in the daily means (67.253, 69.776, and 71.192 mg) is consistent with the random measurement uncertainties of 2.0 mg for all measurements. This indicates that there is a very high probability (1.0 – 1.4% = 98.6%) that something else is also affecting the measurements. This "something else" is the systematic bias, i.e. the 0.264 mg drift in the equipment per measurement that was assumed for this example. This measurement drift was assumed to be 0.264 mg per measurement to obtain a significance level of 1.4%, which is also the value obtained for the C^{14} date measurements for the Shroud samples in 1988 as shown in Table 6 and discussed in Ref. 2.

The conclusion of this example is that the values for the two-sigma range of 69.011 to 71.011 mg should not be regarded as valid because the 1.4% significance level indicates that there is only a 1.4% chance of the measurement data being consistent with the random measurement errors, so that there is a high probability of a systematic bias being present, though other causes may be present as well (non-normality, autocorrelation, and data-dependence). The measurements should not be trusted unless it is possible to determine how much the measurements have been affected by the bias, or unless it can be proven that the measurement uncertainties have been significantly underestimated. In the example above, the bias increased the measured value from the true value of 60.000 mg to the apparent value of 70.011 mg, a change of 10.011 mg. But if the first measurement in Table 2 was measurement number 130 instead of 30, then the average measured value would be 70.011 + (100 x 0.264) = 96.411 mg as in Table 4, a change of 36.411 mg from the correct 60.000 mg, yet the slope in the trendline between the mean values for each day would be the same as in Figure 6. In other words, how much the measurements have been affected by a systematic bias cannot be determined from the slope of the trendline in Figure 6.

Table 5 shows the statistical analysis of the measurements with a bias, both for measurements 30 to 45 in Table 2 and for measurements 130 to 145 in Table 4. Table 5 shows that doing a χ^2

statistical analysis on either set of measurements (30 to 45 or 130 to 145) results in the same χ^2 value (8.5621) and significance level (1.38), even though the final mean changed from 70.011 to 96.411 due to the larger bias. This means that how much the measurements have been affected by a systematic bias cannot be determined from the slope of the trendline in Figure 6 or from a χ^2 statistical analysis as in Table 5. To generalize this statement, if the presence of a systematic bias has been established for a set of measurements, then there is no way to determine the true value from the measurement data. The cause of the systematic bias must be discovered and then quantified by other means, which would often be difficult if not impossible.

The above simplified examples were chosen to simulate the C^{14} dating of the Shroud in 1988, and the conclusions arrived at above also apply to the C^{14} dating of the Shroud. Thus, if it can be established that the C^{14} measurement data indicates that a systematic bias was very likely present, then the measurement data cannot be used to determine the true date of the Shroud. The only exception would be if the systematic bias has been identified and quantified so that the measurement values can be corrected for the bias, which was not done in Damon. The results of a χ^2 statistical analysis of the measurement data from Damon is shown in Table 6, which is from Ref. 2. The significance level of 1.39% on the bottom line in Table 6 for the Shroud is essentially the same as the 1.38% obtained in Table 5. This means that as there was a systematic bias on the measurements in the above example, there is a high probability (~98%) that there was also a systematic bias present in the C^{14} dating of the Shroud in 1988, so that the results should not be accepted as necessarily valid.

To summarize: the C^{14} date measurements of the Shroud in 1988 that produced the corrected date range of 1260 to 1390 AD (two sigma = 95% probability) should not be accepted as necessarily valid because the 1.39% significance level in Table 6 indicates that there is only a 1.4% chance that the spread in the mean values for the three laboratories result only from the random measurement errors that are listed in Damon. This indicates that there is a high probability (98.6%) of a systematic bias being present in the 1988 C^{14} dating.

To avoid the very low significance level (1.39%) obtained in Tables 3, 5, and 6, it is sometimes suggested that the mean should be an unweighted mean based upon the scatter of data instead of a weighted mean. For the example in Table 3 with bias, this does not change the mean values for day 1, 2, or 3 because the uncertainty for each measurement was assumed to be the same (2.000). But is does change the final mean from 70.011 ± 0.500 to 69.407 ± 1.995. This process results in a much higher uncertainty because it is wrapping the effect of the systematic bias into the uncertainty and does not give the correct value of 60.000 mg in the container. Thus, calculating the mean from the scatter of data would not give the correct value or uncertainty if a systematic bias is present. This process was used in Damon to calculate the unweighted mean (690 ± 31) that was used in the rest of the analysis, as well as possibly the mean for Tucson (646 ± 31), as shown for material 1 (samples from the Shroud) in Table 1 of Ref. 3. These errors were the primary cause of the 1988 C^{14} dating of the Shroud to 1260 to 1390 AD.

Differences between the hypothetical example in this section and the 1988 C^{14} dating of the Shroud are the following:

- In this example, the bias was based on the measurement number but in the 1988 C^{14} dating of the Shroud the bias is based on the prior sample location on the Shroud (Ref. 2).
- In this example, each measurement could be plotted by its measurement number as in Figure 5, but in the 1988 C^{14} dating, how the samples sent to each laboratory were subsampled has not been made available and may not have been recorded. This means that a figure like Figure 5 cannot be generated for the 1988 C^{14} dating because the x-axis is the distance from the bottom of the Shroud, but we do not know how the subsamples were cut from the samples, so we do not know their distance from the bottom of the Shroud. But a figure like Figure 6 can be generated for the 1988 C^{14} dating results. This is shown in Figure 7, which is taken from Ref. 2. The similarity of Figure 6 and 7 implies the presence of a systematic bias in the C^{14} dating of the Shroud in 1988.

8. Conclusion

This paper gives the background needed to understand the statistical analysis of the 1988 C^{14} measurement values in Ref. 2 and the neutron absorption hypothesis in Ref. 3. It explains why the C^{14} dating methodology should not be assumed to necessarily be accurate for the special case of the Shroud of Turin and gives an overview of the many reasons that experts on the Shroud have become convinced over the last 30 years that the Shroud of Turin is much older than the range of 1260 to 1390 AD arrived at by C^{14} dating. Material in this paper should be read and understood by the layman before he attempts the more technical discussions in Ref. 2 and Ref. 3. This also applies to individuals with a technical background who may not be familiar with research on the Shroud of Turin. Particular attention should be given to Section 7 which gives a simple example of how a systematic bias can produce wrong measurement values. In this simple example, as in the 1988 C^{14} dating results, unless the bias can be quantified so that the measurements can be corrected, the measurement results should not be accepted as necessarily valid.

9. References

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Biography

Robert (Bob) A. Rucker earned an MS degree in nuclear engineering from the University of Michigan and worked in the nuclear industry for 38 years. He organized the International Conference on the Shroud of Turin (ICST-2017) held July 19-22, 2017, in Pasco, Washington, His website is www.shroudresearch.net . Send comments, questions, or corrections to robertarucker@yahoo.com .

Table 1. Example of Measurements without Bias

Measurement Number	Day	Actual Value	2 dice Total	Delta from 7.0	Measured Value
1	1	60.000	6	-1	59.000
2	1	60.000	8	1	61.000
3	1	60.000	5	-2	58.000
4	2	60.000	7	0	60.000
5	2	60.000	12	5	65.000
6	2	60.000	3	-4	56.000
7	2	60.000	11	4	64.000
8	2	60.000	6	-1	59.000
9	3	60.000	6	-1	59.000
10	3	60.000	6	-1	59.000
11	3	60.000	10	3	63.000
12	3	60.000	8	1	61.000
13	3	60.000	10	3	63.000
14	3	60.000	7	0	60.000
15	3	60.000	7	0	60.000
16	3	60.000	6	-1	59.000
Total				6	966.000
Avg.				0.375	60.375

Table 2. Example of Measurements with Bias

Measurement Number	Day	Actual Value	Random Error	Systematic Bias	Measured Value
30	1	60.000	-1	7.656	66.656
31	1	60.000	1	7.920	68.920
32	1	60.000	-2	8.184	66.184
33	2	60.000	0	8.448	68.448
34	2	60.000	5	8.712	73.712
35	2	60.000	-4	8.976	64.976
36	2	60.000	4	9.240	73.240
37	2	60.000	-1	9.504	68.504
38	3	60.000	-1	9.768	68.768
39	3	60.000	-1	10.032	69.032
40	3	60.000	3	10.296	73.296
41	3	60.000	1	10.560	71.560
42	3	60.000	3	10.824	73.824
43	3	60.000	0	11.088	71.088
44	3	60.000	0	11.352	71.352
45	3	60.000	-1	11.616	70 .616
Total			6	154.176	1120.176
Avg.		60.000	0.375	9.636	70.011

Table 3. Statistical Analysis Without and With Bias

	Without Bias	With Bias
Time	**Individual Measurements, mg of Material**	
Day 1	59.000 ± 2.00	66.656 ± 2.00
	61.000 ± 2.00	68.920 ± 2.00
	58.000 ± 2.00	66.184 ± 2.00
Day 2	60.000 ± 2.00	68.448 ± 2.00
	65.000 ± 2.00	73.712 ± 2.00
	56.000 ± 2.00	64.976 ± 2.00
	64.000 ± 2.00	73.240 ± 2.00
	59.000 ± 2.00	68.504 ± 2.00
Day 3	59.000 ± 2.00	68.768 ± 2.00
	59.000 ± 2.00	69.032 ± 2.00
	63.000 ± 2.00	73.296 ± 2.00
	61.000 ± 2.00	71.560 ± 2.00
	63.000 ± 2.00	73.824 ± 2.00
	60.000 ± 2.00	71.088 ± 2.00
	60.000 ± 2.00	71.352 ± 2.00
	59.000 ± 2.00	70.616 ± 2.00
	Weighted Means, mg	
Day 1	59.333 ± 1.155	67.253 ± 1.155
Day 2	60.800 ± 0.894	69.776 ± 0.894
Day 3	60.500 ± 0.707	71.192 ± 0.707
	Analysis of Inter-day Scatter	
Final mean = weighted mean of weighted means	60.375 ± 0.500	70.011 ± 0.500
χ^2 for weighted means (2 degrees of freedom)	1.0708	8.5621
Significance level* (%)	58.54	1.38

* - The probability of obtaining, by chance, a scatter among the 3 laboratory weighted means as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 4. Example for Measurements 130 to 145

Measurement Number	Day	Actual Value	Random Error	Systematic Bias	Measured Value
130	1	60.000	-1	34.056	93.056
131	1	60.000	1	34.320	95.320
132	1	60.000	-2	34.584	92.584
133	2	60.000	0	34.848	94.848
134	2	60.000	5	35.112	100.112
135	2	60.000	-4	35.376	91.376
136	2	60.000	4	35.640	99.640
137	2	60.000	-1	35.904	94.904
138	3	60.000	-1	36.168	95.168
139	3	60.000	-1	36.432	95.432
140	3	60.000	3	36.696	99.696
141	3	60.000	1	36.960	97.960
142	3	60.000	3	37.224	100.224
143	3	60.000	0	37.488	97.488
144	3	60.000	0	37.752	97.752
145	3	60.000	-1	38.016	97.016
Total			6	576.576	1542.576
Avg.		60.000	0.375	36.036	96.411

Table 5. Statistical Analysis with Bias

	For Measurements 30 to 45	**For Measurements 130 to 145**
Time	**Individual Measurements, mg of Material**	
Day 1	66.656 ± 2.00	93.056 ± 2.00
	68.920 ± 2.00	95.320 ± 2.00
	66.184 ± 2.00	92.584 ± 2.00
Day 2	68.448 ± 2.00	94.848 ± 2.00
	73.712 ± 2.00	100.112 ± 2.00
	64.976 ± 2.00	91.376 ± 2.00
	73.240 ± 2.00	99.640 ± 2.00
	68.504 ± 2.00	94.904 ± 2.00
Day 3	68.768 ± 2.00	95.168 ± 2.00
	69.032 ± 2.00	95.432 ± 2.00
	73.296 ± 2.00	99.696 ± 2.00
	71.560 ± 2.00	97.960 ± 2.00
	73.824 ± 2.00	100.224 ± 2.00
	71.088 ± 2.00	97.488 ± 2.00
	71.352 ± 2.00	97.752 ± 2.00
	70.616 ± 2.00	97.016 ± 2.00
	Weighted Means, mg	
Day 1	67.253 ± 1.155	93.653 ± 1.155
Day 2	69.776 ± 0.894	96.176 ± 0.894
Day 3	71.192 ± 0.707	97.592 ± 0.707
	Analysis of Inter-day Scatter	
Final mean = weighted mean of weighted means	70.011 ± 0.500	96.411 ± 0.500
χ^2 for weighted means (2 degrees of freedom)	8.5621	8.5621
Significance level* (%)	1.38	1.38

* - The probability of obtaining, by chance, a scatter among the 3 laboratory weighted means as high as that observed, assuming the quoted random errors reflect all sources of variation.

Table 6. Recalculated Statistical Analysis of Measurement Data

	Material 1	Material 2	Material 3	Material 4
Source of material:	Shroud of Turin	Linen from tomb at Qasr Ibrim, Egypt	Mummy of Cleopatra from Thebes, Egypt	Cope of St. Louis d'Anjou of France
Expected date:		11^{th} to 12^{th} Century AD	110 BC to 75 AD	1290 to 1310 AD
Laboratory	**Individual Measurements of C^{14} Date, Years Before Present (YBP, Present = 1950)**			
Tucson, Arizona	606 ± 41	922 ± 48	1838 ± 47	724 ± 42
	574 ± 45	986 ± 56	2041 ± 43	778 ± 88
	753 ± 51	829 ± 50	1960 ± 55	764 ± 45
	632 ± 49	996 ± 38	1983 ± 37	602 ± 38
	676 ± 59	894 ± 37	2137 ± 46	825 ± 44
	540 ± 57			
	701 ± 47			
	701 ± 47			
Zurich, Switzerland	733 ± 61	890 ± 59	1984 ± 50	739 ± 63
	722 ± 56	1036 ± 63	1886 ± 48	676 ± 60
	635 ± 57	923 ± 47	1954 ± 50	760 ± 66
	639 ± 45	980 ± 50		646 ± 49
	679 ± 51	904 ± 46		660 ± 46
Oxford, England	795 ± 65	980 ± 55	1955 ± 70	785 ± 50
	730 ± 45	915 ± 55	1975 ± 55	710 ± 40
	745 ± 55	925 ± 45	1990 ± 50	790 ± 45
Laboratory	**Weighted Mean C^{14} Dates (YBP) Based on Above Values**			
Tucson, Arizona	646.52 ± 17.18	927.44 ± 19.70	1995.23 ± 19.89	721.67 ± 20.42
Zurich, Switzerland	676.14 ± 23.74	940.60 ± 23.16	1939.81 ± 28.47	685.16 ± 24.63
Oxford, England	749.17 ± 30.70	937.88 ± 29.43	1977.05 ± 32.71	755.76 ± 25.66
	Analysis of Interlaboratory Scatter			
Unweighted mean of unwt. means (YBP)	695.38 ± 32.15	937.33 ± 6.26	1968.82 ± 14.74	732.16 ± 19.17
Unweighted mean of weighted means (YBP)	690.61 ± 30.51	935.30 ± 4.01	1970.70 ± 16.31	720.86 ± 20.39
Weighted mean of weighted means (YBP)	672.46 ± 12.68	933.98 ± 13.37	1977.05 ± 14.59	720.16 ± 13.40
χ^2 for weighted mean (2 degrees of freedom)	8.55	0.210	2.55	3.95
Significance level* (%)	1.39	90.1	28.0	13.9

* - The probability of obtaining, by chance, a scatter among the 3 laboratory weighted means as high as that observed, assuming the quoted random errors reflect all sources of variation.

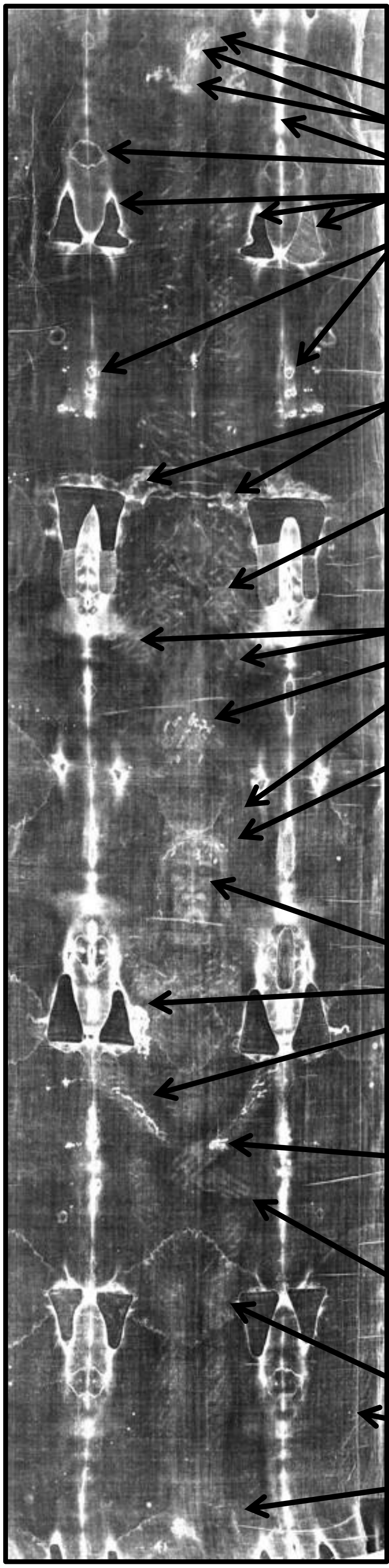

Figure 1. Front and Back Images on the Shroud of Turin

1. Rigor mortis in the feet. This indicates the victim was dead.
2. Two nails through one foot, one of them through both feet.
3. Fire in 1532 resulted in scorch marks and water stains.
4. Areas badly damaged in the fire were patched in 1534.
5. The Hungarian Pray manuscript (1192-1195) has a painting of a famous burial cloth that had long been in Constantinople. It shows the same L-shaped burn holes that are on the Shroud, so the Shroud must have existed significantly (> 2 sigma) before the C^{14} date of 1260 to 1390 AD.
6. The Shroud appears to show a flow of blood and clear blood serum from a wound in the side. Compare with "blood and water" in John 19:34.
7. The Shroud shows 100 to 120 scourge marks from Roman flagrum. Resulting blood marks show blood serum rings (visible only under UV) around the blood exudate. Compare with Mk. 15:15.
8. Abrasions on both shoulders from carrying a rough object.
9. Puncture wounds from sharp objects that pierced his scalp.
10. Pollen on the Shroud unique to the area around Jerusalem. Pollen from a plant with long thorns found around his head.
11. The images are negative images and contain 3D information that indicates the distance of the cloth from the body. Only the top 1 or 2 layers of fibers in a thread are discolored. The discolored fibers in the image result from the carbon atoms that were already in the cellulose molecules in the flax fibers being changed from single to double electron bonds.
12. Swollen cheeks and damaged nose from a beating or a fall.
13. Side wound shows a hole the size of a Roman thrusting spear.
14. Blood running down arms at the correct angles for crucifixion. Blood is real human blood, male, type AB. The blood with high bilirubin content and nanoparticles of creatinine bound to ferritin prove he was severely tortured.
15. All paintings from the Middle Ages show nails through the palms, but this will not support sufficient weight since there is no bone structure above this location. The Shroud shows the correct nail locations - through the wrist instead of the palm.
16. Shroud correctly shows thumbs folded under due to contact of the nail with the main nerve that goes through the wrist. This is also contrary to paintings from the Middle Ages.
17. Abrasions on one knee show a microscopic amount of dirt.
18. Three-inch wide side strip sown on with a unique stitch very similar to that found at Masada (destroyed in 73-74 AD).
19. Microscopic chips of travertine aragonite limestone containing impurities that closely match limestone in Jerusalem.

Figure 2.
Diagram of a Carbon Atom

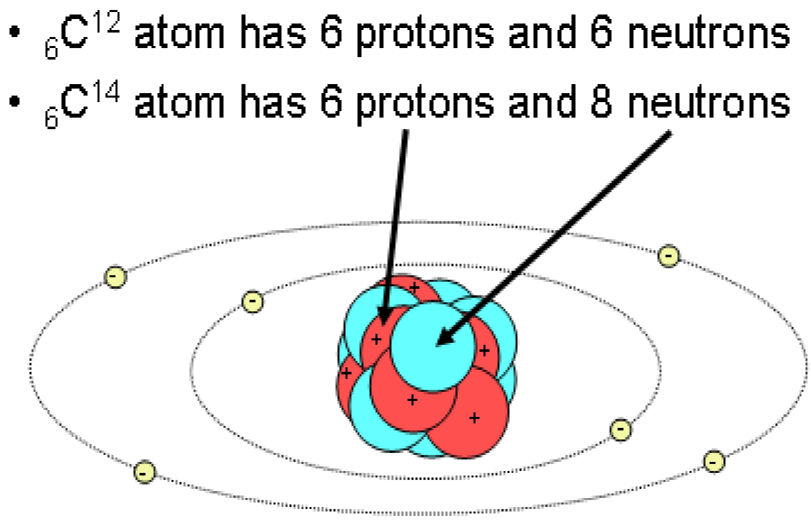

Figure 3. Decay of C^{14} After Death

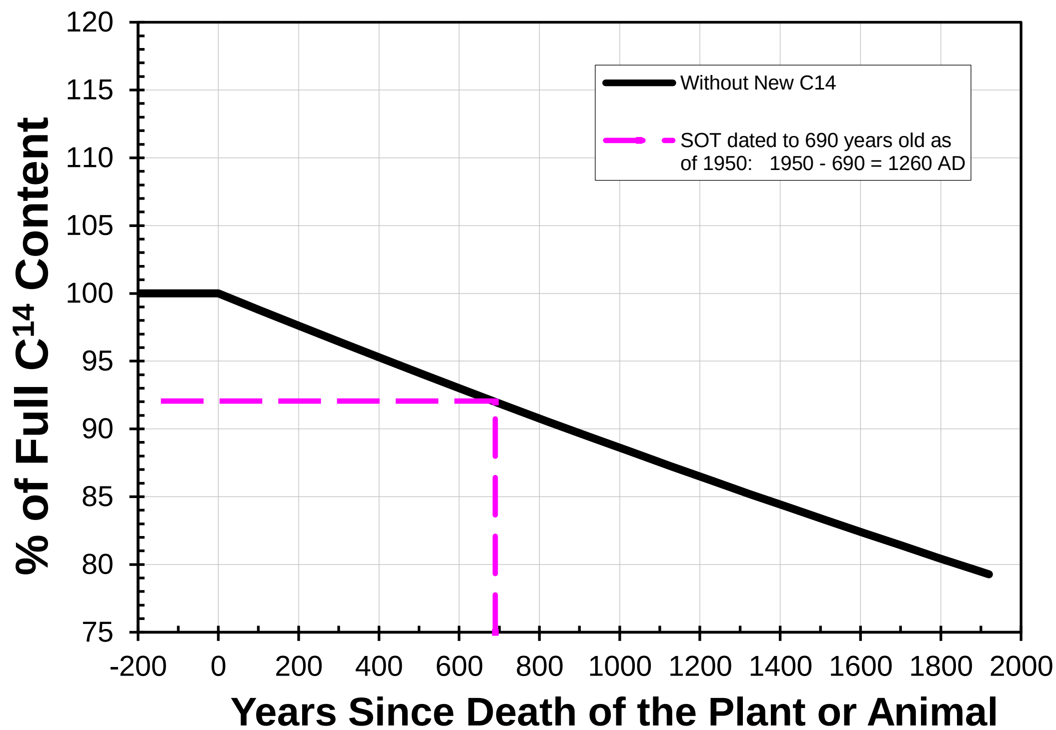

Figure 4. Gaussian or "Normal" Distribution

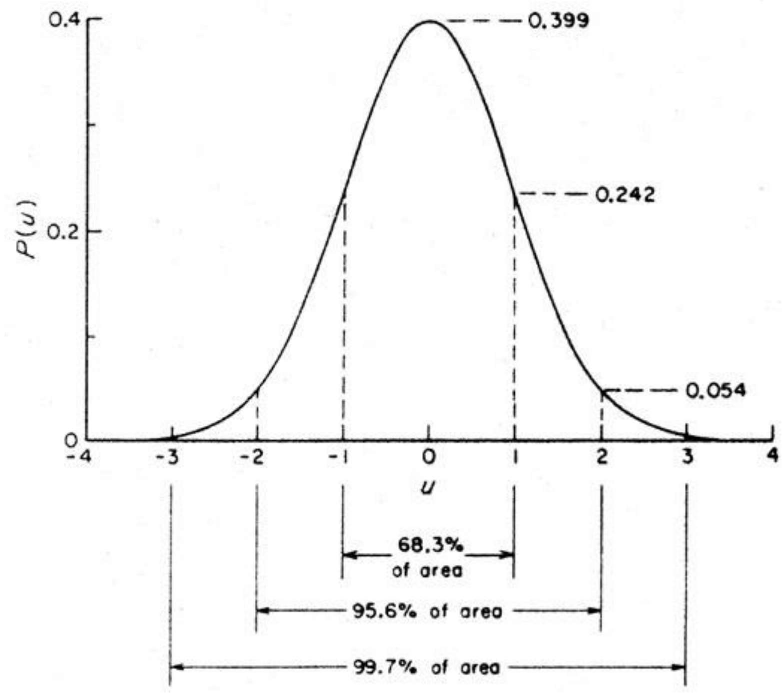

Figure 5. Example of Measurements With and Without Bias

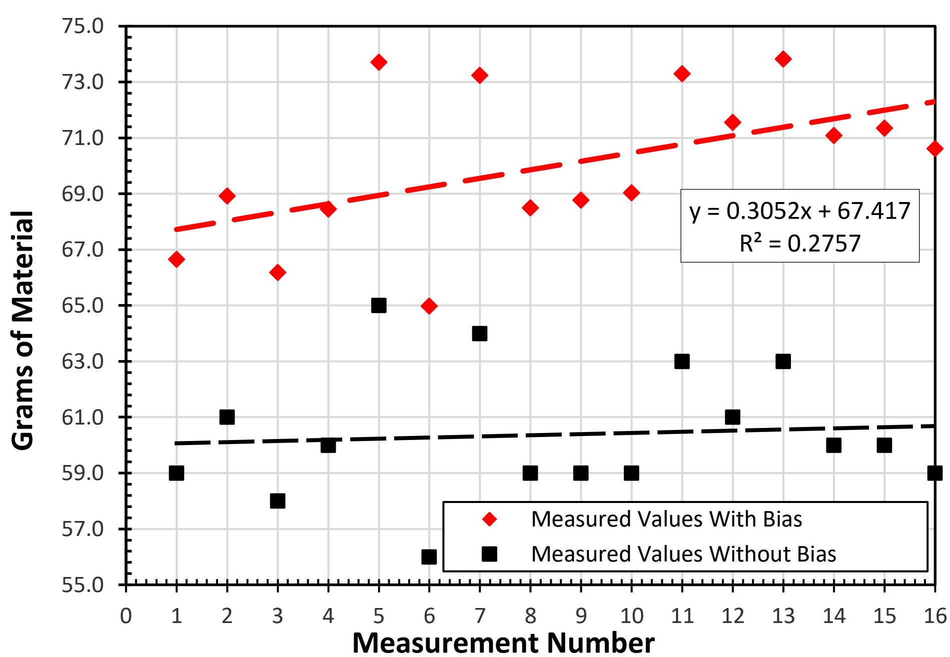

Figure 6. Trendline Between Daily Means With Bias

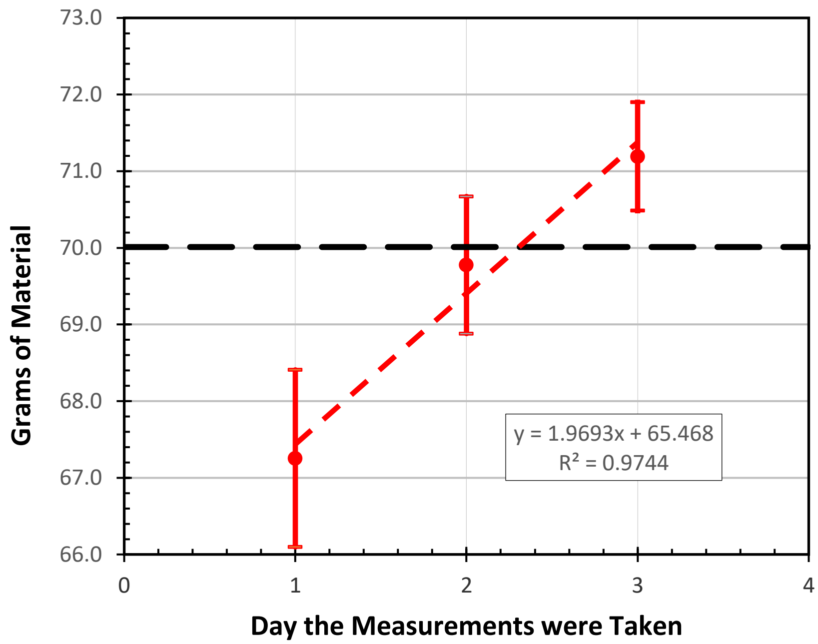

Figure 7. Average C^{14} Date from Each Laboratory, Left to Right: Oxford, Zurich, and Tucson